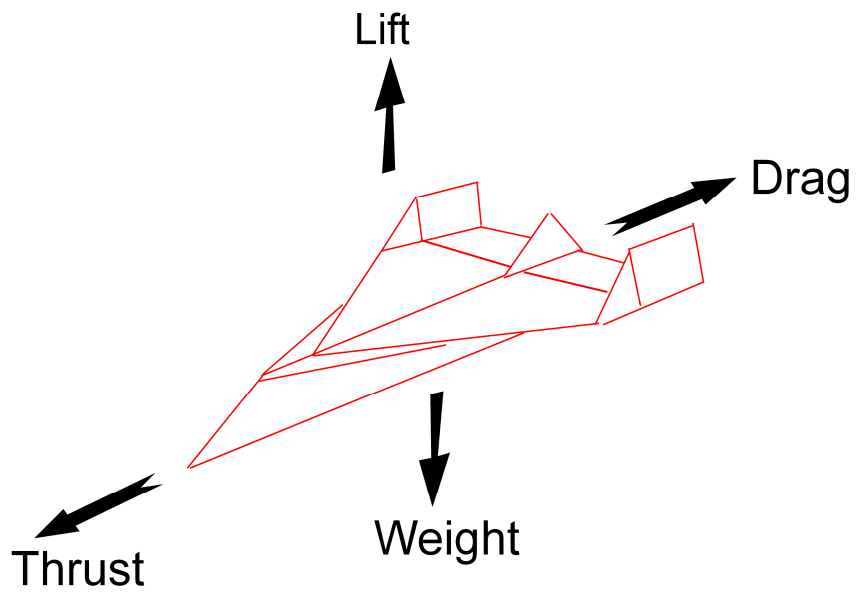


Science of Flight

Introduction to Aerodynamics for the Science Student



Why does a Paper Airplane Fly?

A paper airplane flies because of the scientific properties of gases, which make up our air. The branch of science called *Aerodynamics* is devoted to studying this very question. We are going to start by looking at three properties of gases:

- ✦ Gases are made of very, very small particles called molecules with a lot of space between them (a lot more space than liquids or solids).
- ✦ These particles are not very attached to each other and flow around solid bodies that get in the way.
- ✦ These particles are very energetic having rapid and random motion and collide with each other and everything else that is in contact with the air.

The first property listed is mainly important because it helps to explain the second property. Without having plenty of space between them, air particles could not easily flow around solid bodies that are in the air such as our plane. For example try throwing something under water, which has much less space between molecules, and see how fast it slows down. Also the extremely small size allows the air particles to push evenly and consistently as they have millions of mini collisions with our plane's surfaces.

These collisions are so small that we could not come close to feeling them but are so numerous that a lot of force is exerted on every square inch of surface that has contact with the air. This force is called *air pressure* and is an essential property of air. Air pressure is absolutely necessary to allow airplanes to fly.

There is one very key point to be made about the spacing (actually the average spacing) between molecules. It is measured by calculating the *mass* of molecules in a given volume rather than the spacing between them. This property is called *density*.

Even though density changes due to altitude, temperature, and humidity it is constant in the flow around our plane at any given time. This is true for all planes that are flying at *sub sonic speed* (slower than the speed sound travels in the same air) whether it is a paper airplane or a large (sub sonic) passenger plane. When we study about airflow where the density is constant around our body (plane) we are studying a special branch of aerodynamics called *incompressible aerodynamics*.

Super sonic speed flight is totally different in nature and the following discussions do not apply to planes traveling at the speed of sound or higher. Study of flight at those high speeds is called *compressible aerodynamics* because the density will change over different surfaces of the aircraft.

How does air pressure make my plane fly?

To understand this we must learn a lot more about air pressure. In the last section we talked about air pressure comes from the collisions that the molecules making up air have against a surface in contact with the air. This pressure is dependent on density of the air and on the temperature. This dependency can be stated by the perfect gas law as follows:

$$p = \rho RT$$

p – pressure at any given point
 ρ – density of the air at that point
T – temperature of the air at that point
R – the gas constant

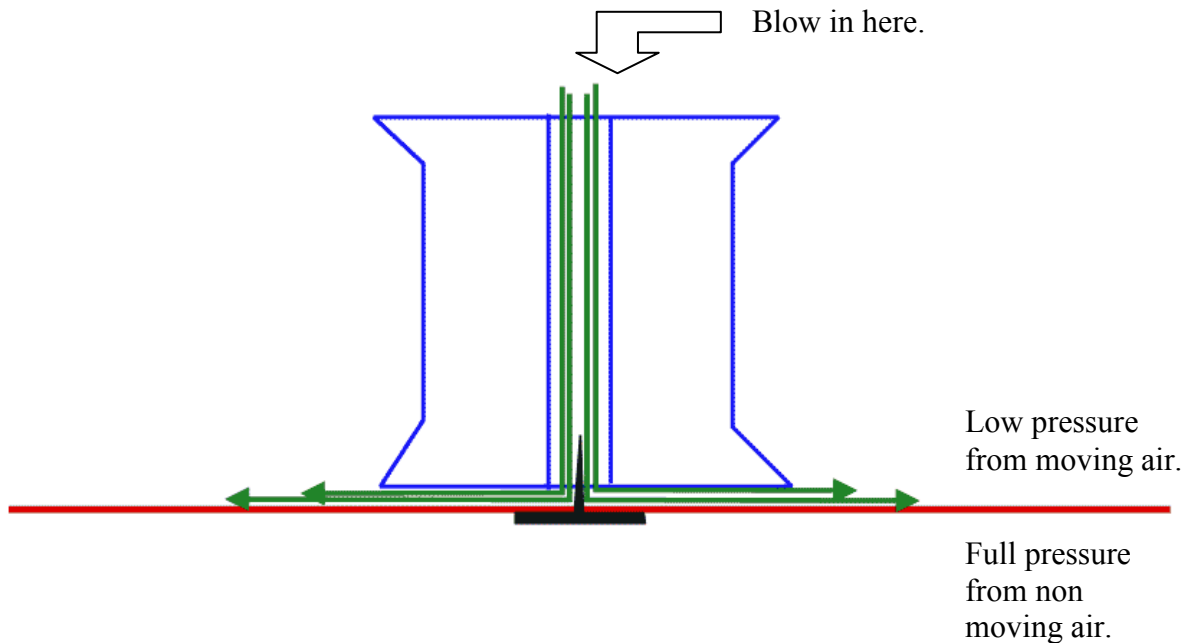
We need to know that under *static* (no motion) conditions air pressure does not exert any *net force* on any object that is surrounded by the air. What this means is that if we held our plane up in the air but were not moving it then air pressure is still acting on it but is *balanced*. This is true because there is the same amount of pushing on the top as on the bottom, the same amount of pushing on the front as on the back, and the same amount of pushing on the left as on the right.

When air starts moving, known as *dynamic*, the amount of pressure that air pushes on surfaces goes down. In fact, the higher the speed, known as *velocity*, that we go the lower the pressure gets. This is expressed as a simplified form of the *Bernoulli Equation*:

$$\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho v_2^2 + p_2$$

ρ – density of the air (constant)
 p – pressure at a given point
 v – velocity of the air at a given point

The easiest way to show that pressure decreases (or pushes less) is with a simple experiment. We start with an index card (or a playing card from an old, no longer used deck) and push a thumbtack or small nail through the very center. Now place a spool of thread (can be an empty spool) on top of the tack or nail. We then pick up and hold the whole assembly together and start blowing through the hole in the top of the spool (see the figure below).



Don't let go of the card until after you have started blowing. The reason for this is that we initially must apply a resistance force to "turn" the flow parallel to the card. After the flow has turned, if we let go of the index card and just hold the spool then the card will stay floating as long as we can keep blowing. But as soon as we stop the card falls to the floor. This happens because the air starts moving parallel to the card and reduces the air pressure on the top. But at the same time there is no motion on the bottom of the card and the pressure is higher. There is no longer a balance of force on the card. In fact, there is enough unbalance to overcome the pull of *gravity* on the card.

While we are on this experiment we are going to sidetrack just a little. Any time motion starts there are things we must do or things that happen as a result of the beginning of the motion that do not continue after the motion is established. These beginning conditions are known as *initial conditions*. Conditions that exist once the motion has passed the beginning and do continue for the rest of the motion are called *steady state conditions*. The effects of initial conditions can be seen on an airplane including paper ones. When we first toss our plane we can sometimes see some wiggling, or a sudden drop, or even a sudden climb then things calm out and our plane starts flying smoothly.

The pressure pushing on the bottom side of our card where there is no motion is called **stagnation pressure** or **total pressure**. The pressure that is pushing against the top surface of the card where there is motion is called the **static pressure**. And the pressure reduction on this surface due to air movement is called **dynamic pressure**. Let's look at how our equation would now look:

$$\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho v_2^2 + p_2$$

$$\frac{1}{2}\rho v_{dynamic}^2 + p_{static} = \frac{1}{2}\cancel{\rho(0)^2} + p_{Total}$$

$$\frac{1}{2}\rho v_{dynamic}^2 + p_{static} = p_{Total}$$

Now some bright student may come up and ask if we can go fast enough to remove all the pressure from our surface. The answer is no and here is why. If we want to find the velocity where there would be no more static pressure than we take the above equation and set the static pressure to zero. Since the total pressure is equal to the atmospheric pressure we plug that value into the equation. We can also look up the density of air and plug it in leaving just speed left over in our equation. Solving for the speed we would get a value higher than the speed of sound and we already have stated that at these values of speed air acts totally different and the density is no longer a constant value. This means the behavior of air changes before we can get fast enough to reduce the static pressure to zero.

Just one last thing about air pressure and we can start talking about airplanes. This is, at least was for me, the hardest to understand. But we need to know that it doesn't matter if the air is moving over the surface or if the surface is moving in the air. If we put a tarp over stuff in the back of a pickup truck and leave it sit in a 50 mile per hour wind or drive down the road at 50 miles per hour on a calm day, that tarp is going to flap just the same. There is going to be the same reduction in pressure on the topside where the air is moving **relative** to the surface of the tarp. This might not be easy to understand, but it means kites fly for the same reasons as airplanes even though the kite stays somewhat still and the wind blows.

We have been on this section for so long that we might need a reminder of the question that got us here. **How does this air pressure make my plane fly?** We might not have answered this one totally yet, but we can think about what if the air moved over the top of our wing faster than over the bottom of the wing. That would cause more static pressure on the bottom pushing up than static pressure on top pushing down. That would give us the lift we need to not fall straight down to the ground. That answer is definitely going to bring up another question.

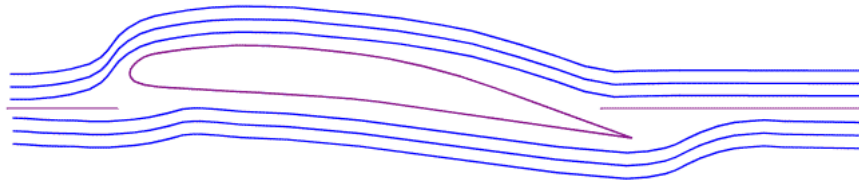
How can air travel faster over my wing than under my wing?

Now we are finally going to get to talk about airplanes or at least wing sections called **airfoils**. Now, if we actually tried to draw the motion of individual air molecules that would be impossible since there is going to be a lot of random motion, remember that third property of gases that we talked about so long ago. What we can draw is the average line of travel for our energetic air particles. With this we will start to have a meaningful diagram. These average lines of travel are called **streamlines**.

Now for a minute lets look at what streamlines would look like over an airfoil if air traveled the same speed on top as on the bottom. Note that in the figure air traveling on the bottom would get to the back edge of the wing first due to the shorter distance it would travel. Since we can't leave a gap with no air on top of the wing the air on the bottom would have to turn the corner and come up to

meet the air traveling over the top of the wing. This is what theory would predict as the motion of an *ideal gas* (not the same as a perfect gas) around an airfoil. An ideal gas is one that has no attraction between the molecules. If this happens then the speed is the same on both top and bottom making the pressure the same on both sides. This leaving no net force or *lift* on our wing. So it won't fly. It's a good thing air isn't ideal.

Air Flow of an Ideal Gas



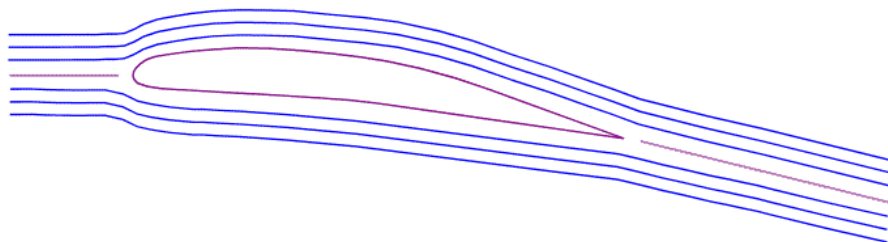
Remember the second important property of air where we said that air molecules were not very attracted to each other? Well “not very” is not the same as “not”. The little bit of attraction we do have between molecules of air is absolutely essential to flight be it birds, bugs, flying squirrels, kites, or airplanes. This attraction between molecules makes air somewhat thick and sticky, a property known as *viscosity*. This stickiness is not near as much as honey or tar but is thick and sticky enough that it refuses to turn a sharp corner.

The proof of this is extremely complex. If we write the mathematical equations for the motion of air particles they are so complex that we can't solve them or at least couldn't when I was in school. These equations are called the *Navier-Stokes equations* and are useful when we can make assumptions and ignore parts of the equations under certain conditions.

But, when it comes to flight we can't theoretically predict what is going to happen. Since we can't solve the problem from theory we must work from observed behavior. And when we mathematically analyze what we observe we develop what is called *empirical* equations as opposed to *theoretical* equations which are derived from proven laws.

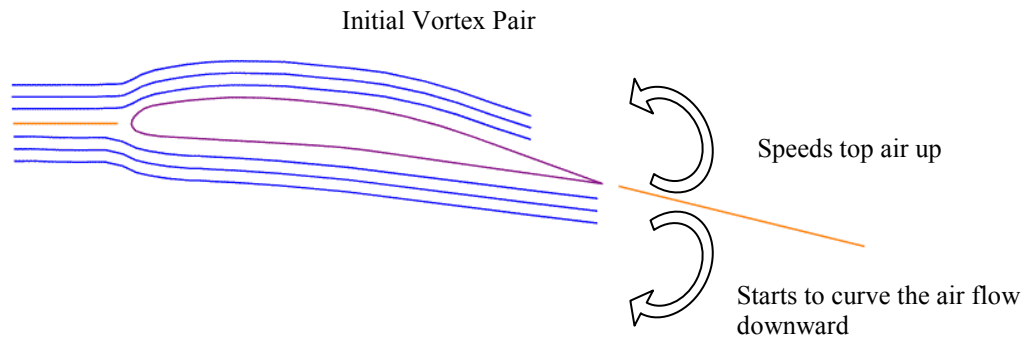
The observed fact that air won't turn a sharp corner is named the *Kutta condition*. It is actually stated as follows: *A body with a sharp trailing edge in motion through a fluid creates about itself a circulation of sufficient strength to hold the rear stagnation point at the trailing edge*. This is a very long way to say that the air won't turn the corner. If we look at the geometry of the airfoil we find that the distance along the top is longer than the distance along the bottom. This will force the top air to go a bit faster than the bottom air. The streamlines once flying at a steady speed are going to look as follows:

Air Flow of a Real Gas



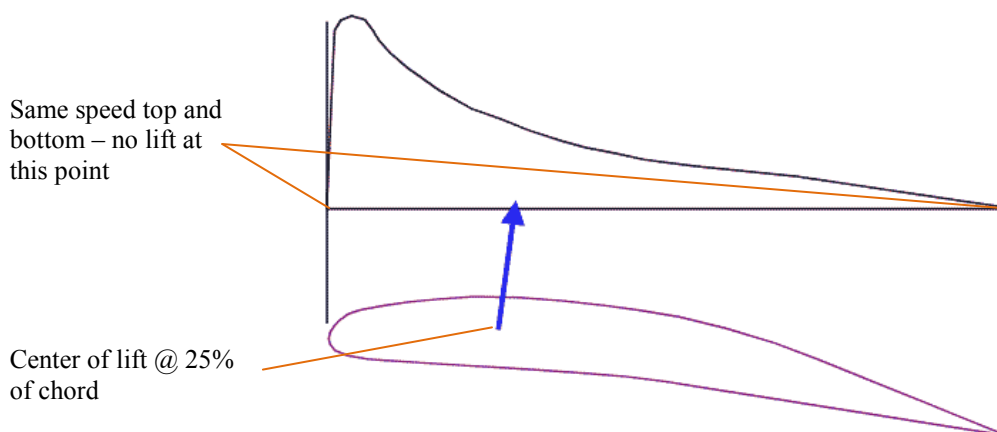
Now if air won't turn the corner what really happens then? Remember we mentioned earlier about initial conditions while discussing our spool experiment? If not, you had better go back up there because we're going to talk about them again. That's because an airfoil in real air starts to act like the ideal gas with air going the same speed on top as on bottom. But as the potential of a

vacuum starts forming at the top rear of the wing due to the fact that the molecules at the top are lagging behind the ones on the bottom there is an initial pair of swirls, called a *vortex pair*, at the rear of the airfoil. This pair of swirls causes the air on top of the wing to speed up and cause the downward slope of the streamlines. Aren't you glad for illustrations because words are hard to use to completely describe what's happening here?



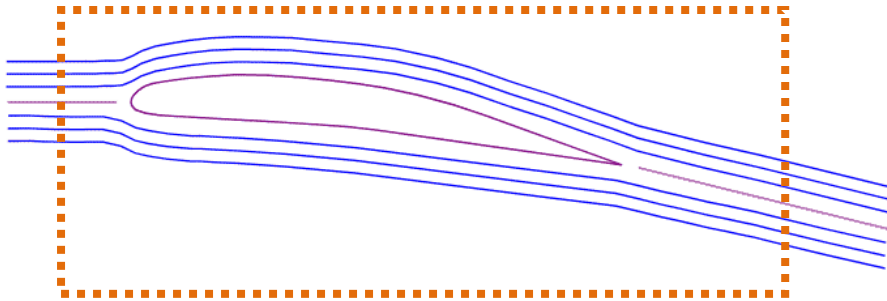
Once the air on top has sped up enough to meet the air on the bottom, lift forms and there is no longer any need for the starting vortex pair. These swirls just get left on the runway or back near our hand for a paper airplane. This starting vortex pair can be seen in movements of dust or smoke for larger aircraft.

There are a few more things that we need to know about the flow around an airfoil. One of those things is that the distribution of lift along the *chord*, or length from leading edge to trailing edge of the wing. At the front of the wing and at the back of the wing the speed of the airflow on top and on bottom must be the same. In order for the air on top of the wing to get to the back at the same time as air from the bottom it must: start at the same speed, speed up faster, and then slow back down to match the bottom air speed. In fact the air on top speeds up, or *accelerates*, very fast on the front portion of the wing chord and slows down, or *decelerates*, much slower as it moves towards the trailing edge of the wing. This means that the lift is not centered on the wing but acts as if it is located one fourth of the way back from the leading edge. The figure below gives an example of the lift distribution along the wing chord:

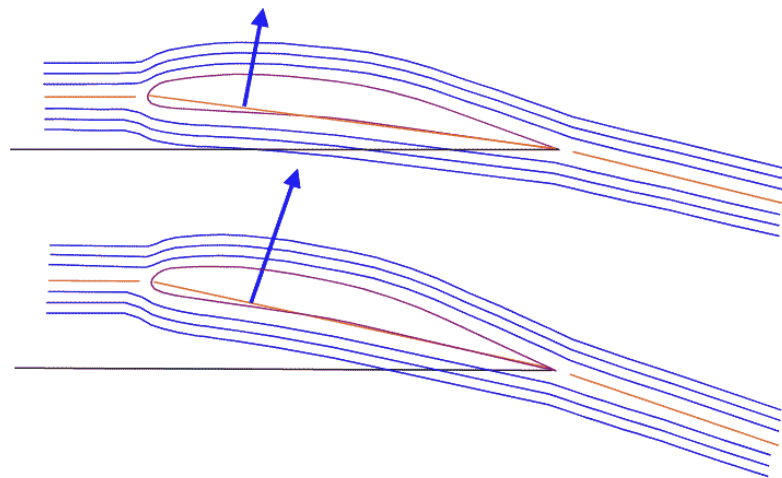


Also, we need to note that as the angle of the chord relative to the horizontal plane, or *angle of attack*, gets bigger the amount of lift goes up – at least at the same speed. To show this let's take a closer look at the illustration for a real gas. When we draw a box that completely surrounds the wing

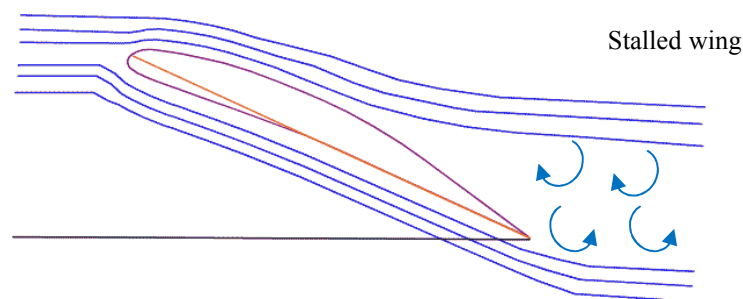
we find that air comes into the box horizontal and leaves angled down. By Newton's Third Law, *for every action there is an opposite and equal reaction*, if the air is pushed down then something, our plane perhaps, is pushed up. It's true the plane was pushed up and an equal force pushed down the air.



Now look at the following figures:



We can note that the lower figure has more angle of attack and that this results in an air flow where the air coming off the trailing edge has a more downward direction. From Newton's Law we can note that a larger action requires a larger reaction or in other words more lift. However, we cannot keep increasing the angle of attack to generate more lift. The reason for this is that we finally reach a point where the air will not follow the surface of the wing from leading edge to trailing edge. The air flow becomes unattached and we lose all lift. This condition is known as **stalling**.



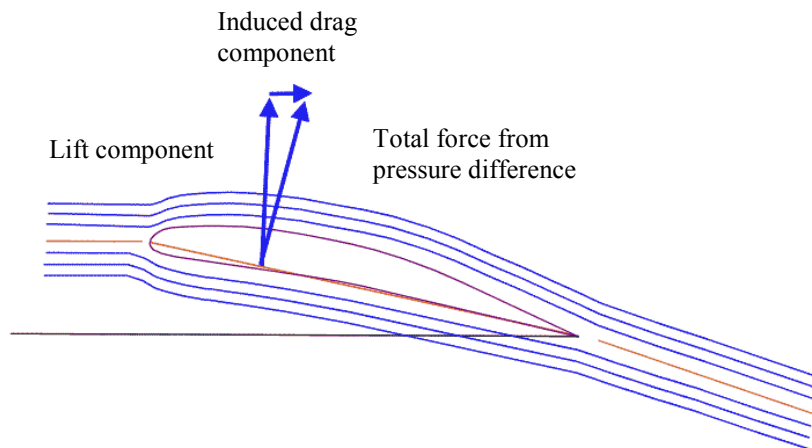
One more thing to know is that the amount of lift is also **proportional** to the speed of the average speed of the air over the wing (speed the plane is moving). Being proportional means that as speed goes up so does the lift. The way we maintain the lift we need to match the weight of the plane as we slow down is to increase the angle of attack. From the above discussion about stall we can see as we

slow down the angle of attack must increase until we reach the angle where we will stall. This is our minimum flight speed.

How much lift we get and this minimum flight speed vary from wing to wing which is why our paper planes work at different throwing speeds. There are many variables that will have effect on these values. These variables include the wing geometry and the texture of the paper. We will get a little more in depth on this subject in our next section.

Last we want to notice one more thing about lift. Lift does not come without adding drag. To fully understand this we must know a little about **vectors**. Vectors are things which are defined by both a **magnitude** (measurement of the amount) and a direction. Remember how we have mentioned both speed and velocity without really defining a difference? Well, the difference is that when we refer to speed we are only concerned about the magnitude but velocity refers to the whole motion including speed and direction of the speed. This makes velocity a vector while speed by itself is a **scalar**, meaning magnitude only.

Getting back to lift we find that it, like all forces, is a vector. If we take another look at the figure of air flow over a wing we can see that the force from pressure differences on the top and bottom surfaces of the wing occurs in a direction perpendicular to the angle of attack. We need to analyze this force to determine how much lift is generated. To analyze a vector we can break it down to smaller components. The components are broken down as to the amount of the lift force that is up and a drag force that slows the plane down. This particular drag force is called **induced drag**. It can be added to the drag from friction forces, called **frictional drag**, of the air on the plane to get the **total drag**. This additional drag is why darts will normally travel further than gliders.



Now we can get back to our question for this section: How can the air travel faster over the top of my wing than under the bottom? Maybe you caught the answer, which is to give the wing a sharp edge at the rear. So we have finally answered the question of how an airplane flies. It should seem that we could stop with the topic of aerodynamics. But there are a few extra topics to cover to be more complete. First someone might just ask:

My wings are flat so why would they act like an airfoil?

The Kutta condition is actually going to make air flow over a flat plate act very much like it would flow over an airfoil. The truth is that an airfoil is simply a modified flat plate that is designed increase lift and to reduce frictional drag over the wing. To understand how air travels further over the top of a flat plate than on the bottom requires us to bring up another topic. This is definitely one of the most important topics in aerodynamics – **boundary layers**.

In the last section we mentioned that how wing geometry and paper texture both had an effect on the lift, drag, and minimum flight speed. How these variables effect the lift requires the analysis of what is happening at the air that is very close to the surface of the wing where all the friction

between wing and air is occurring. This is the boundary layer and its study is too complex to completely cover here. But, we are going to look at some of the characteristics of the boundary layer.

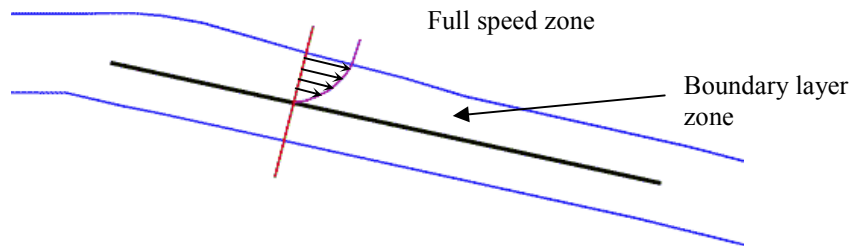
We need to make one quick point here, and that is that there are two types of incompressible flow: laminar and turbulent. Laminar is a very smooth flow where the air travels along streamline with very little deviation. Turbulent flow is very "choppy" and does not consistently follow a steady streamline. As an example think of a sheet hanging on a clothes line; a slow breeze will cause it to rise smoothly but a heavy wind makes it flap violently. This is the difference between laminar and turbulent. Both conditions will produce lift and allow flight but laminar flow produces much less friction and therefore has less drag. We can predict when flow will change from laminar to turbulent by looking at the **Reynolds number**. The Reynolds number for any given wing flow is calculated as follows:

$$R = vl\rho/\mu$$

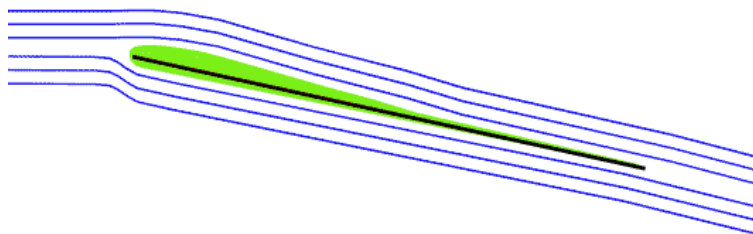
- R – Reynolds number at a point
- v – velocity of the air at that point
- l – chord length of the wing
- ρ – density of the air
- μ – viscosity of the air

For a given surface geometry and texture there is a Reynolds number that will cause flow to change from laminar to turbulent. With the low speeds of paper airplanes our Reynolds number will stay low and the flow remain laminar. This is the only condition we are going to study.

Getting back to our study of boundary layers, we need to know exactly what a boundary layer is and what causes it. We are going back to our property of viscosity, that little bit of stickiness air molecules have for each other. The air molecules also have stickiness to our paper. This causes the air molecules next to the surface of our paper wing to stick to the wing so that they have almost no velocity relative to the wing. Just above these are molecules that have just a tiny bit more speed than the first layer. One more layer up and we are moving just a little faster. Eventually, we will get to molecules that are moving full speed of the flow relative to the wing. The air between the wing surface and this first layer of full speed air is called the boundary layer. It can vary in thickness depending on many factors including the speed the plane is moving. The boundary layer may be a couple of inches thick for a full size plane but only a fraction of an inch for a paper plane. We can visualize the speed of each location in the boundary layer as shown:

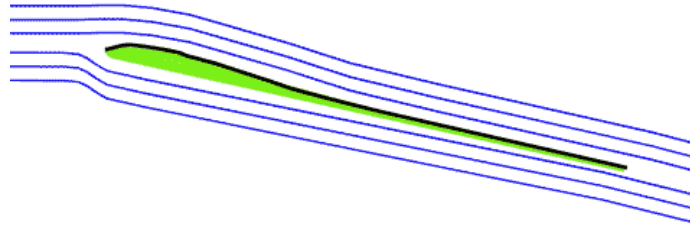


From here we want to look at the shape formed when we plot the end of the boundary layer at each cross section. The thickness of the boundary layer as we travel down the length of the wing changes. In this figure the boundary layer is shown in green. What we can start to see is that the outer limit of the boundary layer takes on the shape of an airfoil. The streamlines act as if they are moving around a solid that has the shape of the boundary layer. This is a **virtual** airfoil.

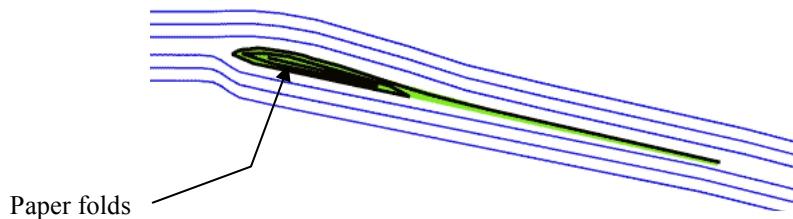


An interesting point is that the pressure on the wing is the same value as the static pressure in the full speed flow. This can happen because even in incompressible flow we can't make air more dense but we can cause the density in the boundary layer to become reduced by the amount necessary to make the total pressure at the surface equal the static pressure of the flow.

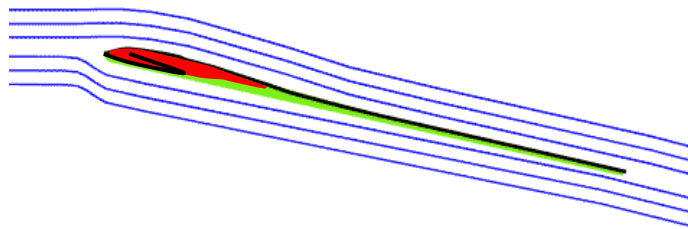
When our flat plate is made of paper it is flexible and will bend under pressure to produce a virtual airfoil more like this one:



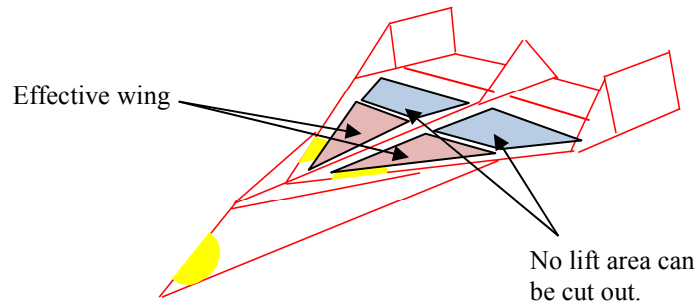
The thicker boundary layer that is formed near the leading edge causes extra lift but we still have a lot of friction drag, although maybe not as much as a flat sheet. We can improve our flight time by lowering this drag by folding the paper so that we "fill in" some of the virtual shape as shown in the next figure.



There are conditions where some air gets "trapped" and the boundary layer is between this trapped air and the full flow. (This is known as an eddy and another example can be seen in water when there is no current directly downstream of a large rock in a fast flowing stream.) This trapped air may circulate around in its "pocket" but stays put as the wing travels. The following will show an eddy in red and the boundary in green

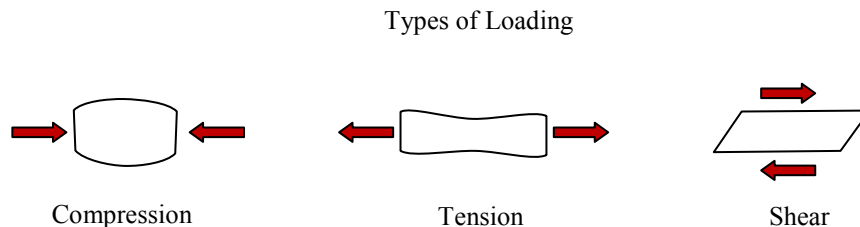


The length of the virtual airfoil does not exist the whole length of the wing. This basically means that only the front section of the wing is acting to produce lift. Back in the 1960's it was a popular idea to cut out the no lift area to reduce weight and drag. I don't think a whole lot of improvement in flight was made but the planes did look more like full size planes.



Note: we can reduce drag by eliminating the wasted area of our plane that makes up the body. As we eliminate the body and create folds to produce an airfoil shape our planes will fly longer and slower. This may be good in a competition but we find that good target shooting ability is lost. That is why I prefer faster type gliders.

Before we completely finish with boundary layers we want to consider why this layer is so critical in determining how much frictional drag is produced as air moves across our wing. To start we need to understand about shear. There are three ways that we can apply equal but opposing forces into a body. The forces can squeeze, squash, or warp. These actions are called compression, tension, and shear. These can be illustrated as follows:



Within the boundary layer each individual layer of air is being sheared between the layer below and the layer above. This shearing produces the different speeds between layers as individual air particles slip across each other. As the particles slip over each other there is an attraction between them to resist this slippage. As the particles overcome this resistance they lose dynamic energy in the direction of flow. This loss of energy is in the form of a force, friction, acting through the length of the boundary layer. A student of physics will recognize that any force acting through a distance is work. And that work can change energy of motion (kinetic energy) into heat energy.

A major objective for an engineer is to reduce this energy loss by reducing the friction force. One of the ways to accomplish this goal is by thinning the boundary layer and making it a more consistent thickness along the whole chord of the wing. This is accomplished when possible by using an airfoil as opposed to a flat plate.

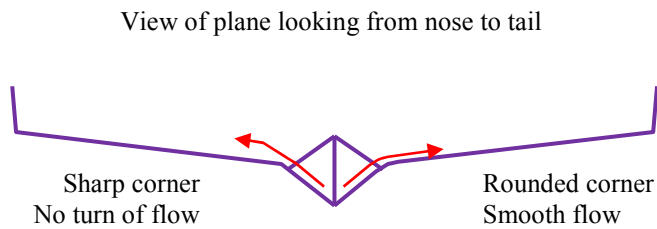
Why do some of my paper airplanes fly poorly?

One of the worst things we can do is have a paper that is not very flexible. As we were talking about boundary layers above we probably should look at what is happening to the lift and drag. A very important characteristic of a paper airplane is the lift to drag ratio. Since we only have our initial toss to get us as far as possible, it becomes very important that we have a good lift to drag ratio (a ratio is the value we get by taking the first number and dividing it by the second number). In terms of boundary layers each of the following steps is an improvement. Our paper:

1. Acts like a flat sheet.
2. Curves to shape of the boundary layer.
3. Forms an eddy.
4. Fills in the gap to make an airfoil.

To get from step 2 to step 4 we design the number and geometry of the folds. But to get past step 1 we must have a good flexible paper to make a good paper airplane. If our paper is stiff we lose some of our lift to drag ratio and our paper airplane will not fly as far. If it is too stiff it may curve more on one side than the other and our wings become unbalanced and the plane will roll or even go into a spin and not fly at all.

Next we are going to look at one more effect sharp corners can have on our paper airplane. If you read the section on trimming you may have noticed that it was mentioned that if you did not use tape then you may have to smooth out the edges between the plane's body and the wings. The reason for this is that if the plane has a tail then the tail will act like a plow and force the air coming down the body channel to flow up and over the wing as shown below. If there is a sharp corner between the body and the wing then the air will not turn the corner smoothly. If the air does not turn smoothly it will cause poor lift over much of the wing and the plane will fall.

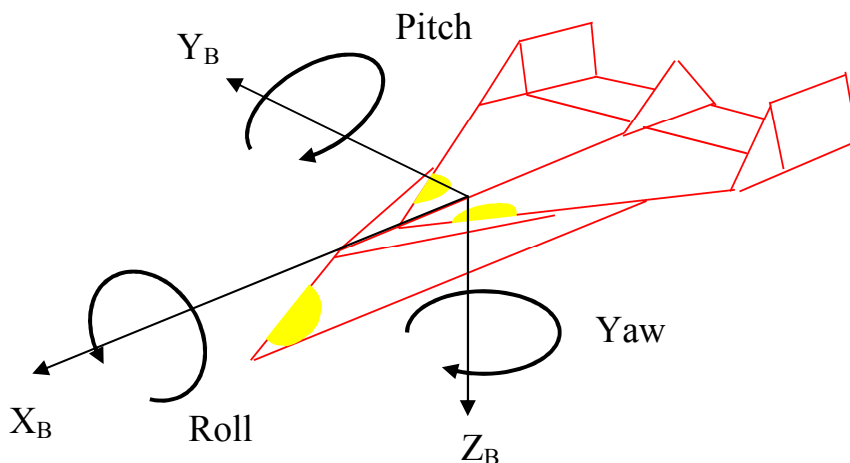


It does not take much of a roundness to allow smooth flow. This can also be a real problem if one edge is rounded enough for smooth flow but the opposite edge is not. This causes unbalanced lift and the plane will begin to roll making the plane unstable. This instability brings up another interesting question:

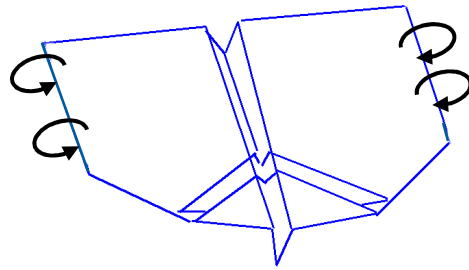
What are some other things that can make my plane unstable?

If a small change being made to the *orientation* of our airplane causes a larger change to occur than we have an *unstable* condition. However, if a small change in the orientation causes the plane to restore itself or at least to resist additional change to the original orientation than we have a *stable* condition. To understand what we are referring to here we must know what orientation means. Orientation is the direction our plane is pointing and a change to the orientation requires a rotation.

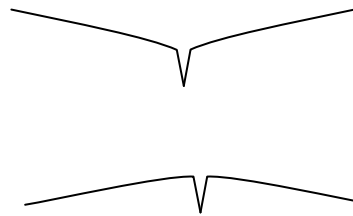
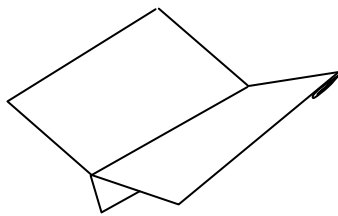
This leads us to look at the axis system we use to define our plane. This system is called the body-axis system and has six degrees of freedom as shown below:



The instability caused by not smoothing the folds is in the Roll direction. Another cause of roll instability is if we need fins on our wings to keep from air flowing off our wing side ways as shown below. This action is dependent on the ratio of wing span to the chord length known as the *aspect ratio*.



There is one more condition that can cause roll instability and has happened in badly designed full size aircraft. This one is not easy to follow but can be deadly. It is instability due to negative *dihedral angle*, the angle the wing makes from horizontal.



Positive dihedral
Stable condition

Negative dihedral
Unstable condition

To visualize this type of instability make two basic box planes. For one plane fold the wings with a positive dihedral and the other with a negative dihedral. Now start with the positive one. Hold it in front of you and pointed away. Move the tail to the left and nose to the right, which is a slight rotation in the yaw axis. This can happen from a gust of wind hitting the tail. Just remember that the plane is still traveling away from you and no longer traveling the direction it points. If you look very carefully, you will notice that this makes the air come from a lower angle to the left wing increasing the wings angle of attack and the lift on the right wing. This extra lift causes the plane to roll making the left wing go up. That in turn increases the right wing angle of attack. The wings balance their lift, the plane banks to the right and will level off. We are now headed a new direction but no danger occurred. This is a stable reaction.

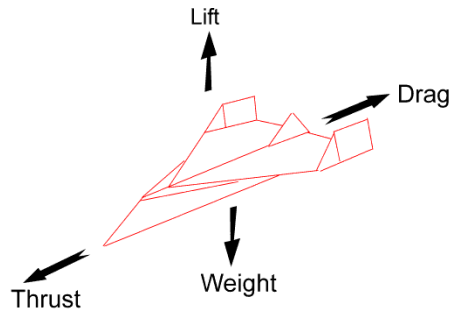
Now try the same thing with the negative dihedral wing. Move the tail to the left and the nose to the right. The air now flows more from the top of the left wing lowering the angle of attack and lift on the left wing. This will cause the plane to roll left wing down, right wing up. This in turn makes the angle of attack even worse for the left wing and the problem just gets bigger. Very quickly the plane will be rolling counterclockwise as seen from the rear. Lift will no longer occur on either wing and the plane will spin out of control towards the ground. This instability is known as the Dutchman's roll and has killed many pilots.

Instability in the pitch axis is caused by the lift not being in line with the paper airplanes *center of gravity*, or balance point. There is little that can be done other than to adjust the design to change the planes center of gravity. Likewise, for a paper plane instability in the yaw direction is a design flaw due to the wing stalling. This usually occurs when the plane is moving too slow to produce enough lift to keep in the air. Often it is caused by the center of gravity being too far back and making the nose pitch upward.

From here we may want to consider the following:

What are some additional equations I should know about paper airplane dynamics?

Remember the sketch below from our earlier discussion of airplane loads? A few of the laws of physics that would help analysis the effect of these loads on our plane are: *Newton's Laws*, *Conservation of Energy*, and *Conservation of Momentum*.



We will start with Sir Isaac Newton's three laws of motion and Law of Gravity:

First Law – If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

Newton's First Law basically states that we must apply a force to start our airplane flying. This may seem quite obvious at first but once we start looking that it also states that any change in motion requires a force. This implies that since we can observe our paper airplane slow down than we can conclude a force is working on it, specifically we call this force drag. This law also shows the importance of direction implying that force and straight line speed (velocity) are vector quantities.

Second Law – If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

The second law is an extension of the first law and gives us the basics for generating an actual equation comparing the amount of force to the rate of change of the straight line speed (acceleration). It adds the concept of a constant of proportionality as a property of the particle. This constant of proportionality is known as mass. We can measure it by finding the amount of force required to balance it under the influence of gravity. This balancing force is known as weight. This law can be written by the equation:

$$F = ma$$

F – force on the particle
m – mass of the particle
a – acceleration of the particle

Third Law – The forces of action and reaction between bodies in contact have the same magnitude, same line of action and opposite sense.

The main ideas we will pick up here in terms of paper airplanes is that in order for air to "push" a paper airplane up (lift) then the paper airplane must equally push the air down. The section on airflow over a wing demonstrates this concept. Another use for this law, in understanding full size airplanes, is that to maintain speed we must push on the air the same as the air pushes back. This is why an engine can push against the air in order to generate thrust.

Newton's Law of Gravitation – Two particles of mass M and m are mutually attracted with equal and opposite forces F and –F of magnitude F given by the formula:

$$F = G \frac{Mm}{r^2}$$

F – force of attraction on each particle
M – mass of the larger particle
m – mass of the smaller particle
r – distance between particles
G – gravitational constant

Newton's Law of Gravitation tells us that gravity is pulling our airplane down. It allows us to calculate how much lift we need to generate in order to oppose gravity. We often talk about the acceleration of gravity. If we take the above equation and substitute mass (of our airplane – "little" m) times acceleration for force and then divided both sides by "little" m we get the following:

$$a = G \frac{M}{r^2}$$

If we are anywhere near the Earth's surface the value of r^2 is constant as is the mass of the Earth, M, and G, the gravitational constant. This means that acceleration is constant and can be calculated to be 32.17 ft/sec².

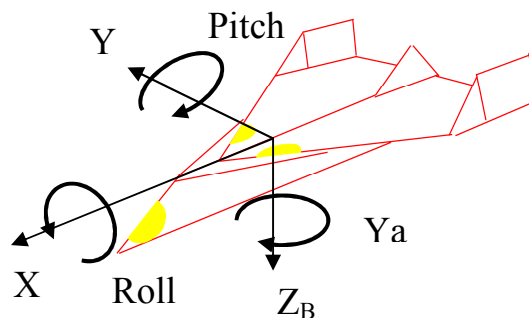
The next topic we need to understand is about moments and rotation. Above we talked about forces that were in a straight line of action with each other. But, what happens when the forces are not in a straight line? The answer is that we find that there is an equivalent set of laws of motion that deal with a body as opposed to a particle. The difference between a particle and a body is that a body has size where a particle does not. A particle is only a theoretical object since all solids in the real world have size.

Any misalignment between forces is called a moment and a moment will cause an equivalent to speed, called angular velocity (represented by the symbol ω), to change in value. The rate of this change is called angular acceleration. The equation for this relationship is:

$$M = \bar{I}\alpha$$

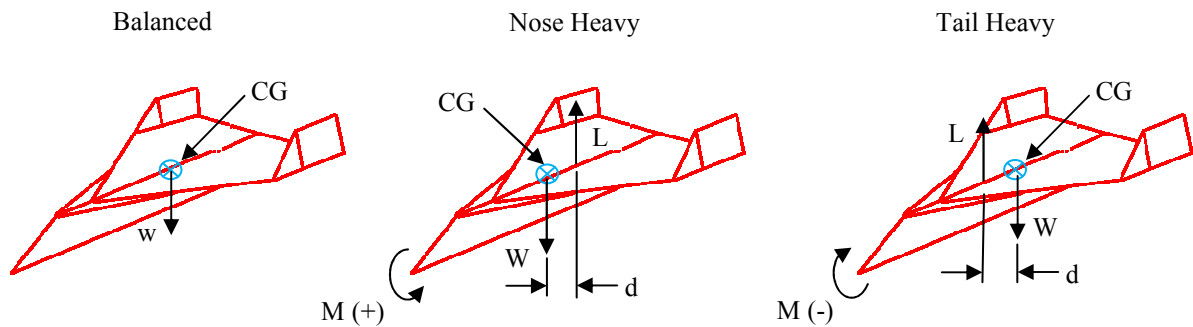
M – moment applied to the body
 \bar{I} – moment of inertia of the body (resistance to change in spin)
 α – angular acceleration

For our purposes we are not going to concentrate on this equation too hard. What we do want to understand is that if our forces are not lined up this will cause our paper airplane to spin and that this is due to the moment that is acting on the body. We do want to learn how to calculate the amount of moment a body sees. Remember the illustration below? We are going to learn how to calculate the value of the moment that would produce Pitch.



We are going to diagram the forces that act in the X – Z plane, this means the two dimensional space that would include both the X axis and the Z axis. We are also going to make the assumptions that the thrust is zero (since our body is a flying paper airplane) and that the drag acts in line with the **Center of Gravity** (or CG). Before we go any further we must define CG. It was mentioned above that a real body differed from a particle since it had size. It can also be noted that all bodies have a

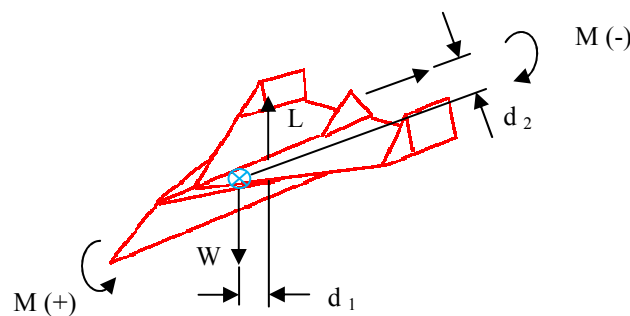
balancing point where we can apply a force without causing any spin. This is the point known as the CG and happens to be the same point that the bodies weight will act through. All of this said we can diagram our forces that we have left on our paper airplane:



To calculate the moment we need to know the value of the forces and their distances (d) from the CG. There are two sets in our current calculation: weight (w) at 0 distance and lift (l) at distance d. If the force would turn the body clockwise around the CG it is negative and if it would turn the body counterclockwise it is positive (this is called the right hand rule). To get the value of the moment we add all of the sums of the products of the forces and distances. Using the symbol Σ to mean sum of all, then our equation for moment becomes:

$$m = \Sigma(fd)$$

Note that for the above examples the moment is equal to the lift times the distance it is off from the CG since the weight always acts through the CG and produces zero moment. If we analyze the trimming of a paper airplane we will see more forces involved since we will add the effects of a tail or of a trimming adjustment that acts like a tail. We can now see how in figures 2 and 3 we must add an additional force to get a moment that will cancel out the moment produced by the misalignment of the weight and the lift. This following example will show how folding a tail balances a nose heavy paper airplane.



We can also analyze the dynamics of our flight by using either work-energy or impulse-momentum methods. To do this we must learn a little more Physics. If we have a force which pulls on a body as that body moves a distant d then this force does work on the body. The value of this work is the product of the force and the distance it pulled the body through. This work will cause a change in the energy of the body which is equal to $\frac{1}{2}$ of the mass times the speed squared. Let's take a look at this in the form of equations:

$$Work(W) = Fd \text{ and } Energy(E) = 1/2(mv^2)$$

Using the symbol Δ to mean "change of", or the amount of change in, then the laws of Physics tell us that the following equation is true.

$$\text{(Work)} \quad Fd = 1/2(m(\Delta v^2)) \quad \text{(Energy)}$$

This relationship is known as the work-energy relationship. It is interesting to note that if we push very hard on a wall that does not move then we are not doing any work!

The next relationship we need to know is the impulse-momentum relationship. Impulse is defined as the product of a force, acting on a body, and the amount of time it acts on the body. Momentum is defined as the product of a body's mass and its speed of motion. Physics once again tells us that:

$$\text{Impulse } (I) = Ft \quad \text{and} \quad \text{Momentum}(M) = mv$$

Then

$$\text{(Impulse)} \quad Ft = \Delta mv \quad \text{(Momentum)}$$

Examples of how these are used can be:

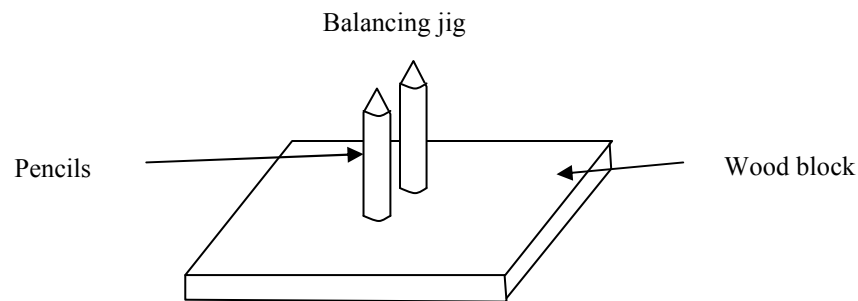
$$\text{Max time possible for flight with force of Drag: } (Drag)t = \Delta mv \quad \text{or} \quad t_{max} = \frac{\Delta mv}{(Drag)}$$

$$\text{Max distance possible with force of Drag: } (Drag)d = \frac{1}{2}m\Delta v^2 \quad \text{or} \quad d_{max} = \frac{\frac{1}{2}m\Delta v^2}{(Drag)}$$

How can I use paper airplanes in a Science Fair?

Paper planes can be used for many types of science fair projects. Here are just a few suggestions:

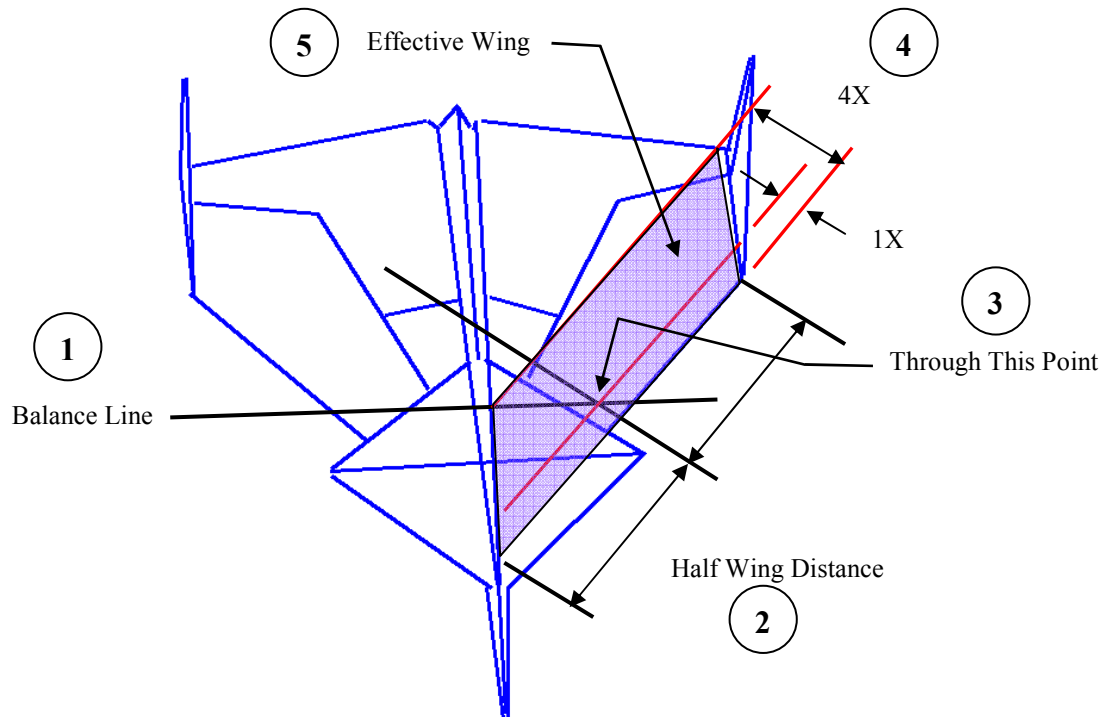
1. Make a balancing jig (see figure below) and find where the center of gravity is for different plane designs. Using the fact that this should also line up with the spot 25% of the chord length you can figure out how much of the wing is producing the lift.
2. Build different designs and check the effective wing from above. Calculate the aspect ratio and determine if it has any effect on the planes need for a tail or for fins.
3. Use different weights and textures of paper and record the effects on flight time and/or distance.
4. Chose a design that will allow small dimensional variation. Make several and tweak in the dimensions to optimize the design.
5. Build several planes of the same model using the same type and weight of paper. Record several flights for each and determine if this design is consistent or varies a lot from plane to plane. Try to reason causes for variation. Note: some planes are very temperamental from plane to plane while others are not.



Whether you choose one of these projects or think of your own make sure to take very organized notes and record it neatly. Use graphs and charts when they help convey your conclusions. Stick to the scientific method and present your background research. Using these suggestions the student should be able to present a very good science project. This might make us wonder what more do we need to know for a good presentation.

How do I find the center of lift of a paper airplane?

Since finding the effective wing is important for several projects and many paper airplanes have wings that sweep back, here is how to figure the 25% point (known as the quarter chord). First, find the balance line and sketch it on the wing. Measure halfway down the length of the wing and draw a line perpendicular to the leading edge of the wing through this halfway point. Where this line crosses the balance line is the 25% line. Next draw a line through this point parallel with leading edge of the wing. Draw another line four times this distance from the leading edge. This will mark the approximate boundary of the effective wing



Remember Science Should Be Fun!

Pat Morgan