Section 4
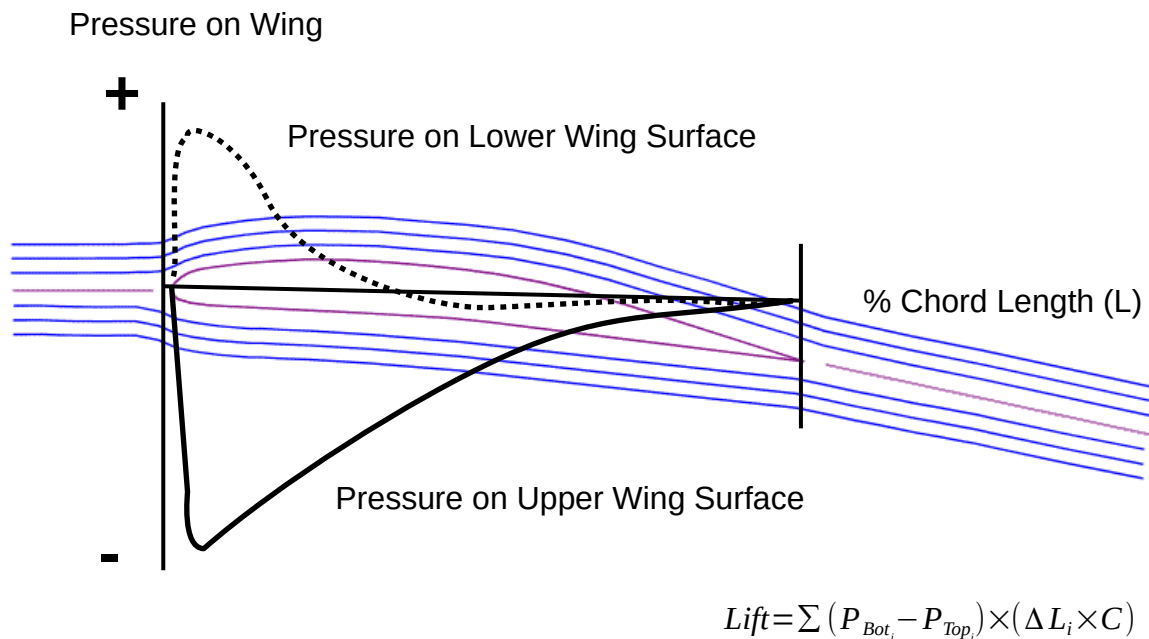# Science of Flight

Introduction to Aerodynamics for the Science Student

Pressure on Wing

+

Pressure on Lower Wing Surface

% Chord Length (L)

Pressure on Upper Wing Surface

-

$$Lift = \sum \left(P_{Bot_i} - P_{Top_i} \right) \times \left(\Delta L_i \times C \right)$$

Why Does a Paper Airplane Fly?

A paper airplane flies because of the scientific properties of gases, which make up our air. The branch of science called ***Aerodynamics*** is devoted to studying this very question. We are going to start by looking at three properties of gases:

- Gases are made of very, very small particles called molecules with a lot of space between them (a lot more space than liquids or solids).
- These particles are not very attached to each other and flow around solid bodies that get in the way.
- These particles are very energetic, having rapid and random motion, and collide with each other and everything else that is in contact with the air.

The first property listed is mainly important because it helps to explain the second property. Without having plenty of space between them, air particles could not easily flow around solid bodies that are in the air such as our plane. For example try throwing something under water, which has much less space between molecules, and see how fast it slows down. Also the extremely small size allows the air particles to push evenly and consistently as they have millions of mini collisions with our plane's surfaces.

These collisions are so small that we could not come close to feeling them but are so numerous that a lot of force is exerted on every square inch of surface that has contact with the air. This force is called ***air pressure*** and is an essential property of air. Air pressure is absolutely necessary to allow airplanes to fly.

There is one very key point to be made about the spacing (actually the average spacing) between molecules. It is measured by calculating the ***mass*** of molecules in a given volume rather than the spacing between them. This property is called ***density***.

Even though density changes due to altitude, temperature, and humidity it is constant in the flow around our plane at any given time. This is true for all planes that are flying at ***sub sonic speed*** (slower than the speed sound travels in the same air) whether it is a paper airplane or a large (sub sonic) passenger plane. When we study about airflow where the density is constant around our body (plane), we are studying a special branch of aerodynamics called ***incompressible aerodynamics***.

Super sonic speed flight is totally different in nature and the following discussions do not apply to planes traveling at the speed of sound or higher. Study of flight at those high speeds is called ***compressible aerodynamics*** because the density will change over different surfaces of the aircraft.

How Does Air Pressure Make My Plane Fly?

    To understand this we must learn a lot more about air pressure. In the last section we talked about how air pressure comes from the collisions that the molecules making up air have against a surface in contact with the air. This pressure is dependent on density of the air (how many millions of collisions per square inch) and on the **temperature** (how much energy or how hard is each collision). This dependency can be stated by the perfect gas law as follows:

$$p = \rho\, R T$$

|	
p	– pressure at any given point
ρ	– density of the air at that point
T	– temperature of the air at that point
R	– the gas constant

    We need to know that under **static** (no motion) conditions air pressure does not exert any **net force** on any object that is surrounded by the air. What this means is that if we held our plane up in the air but were not moving it then air pressure is still acting on it but is **balanced**. This is true because there is the same amount of pushing on the top as on the bottom, the same amount of pushing on the front as on the back, and the same amount of pushing on the left as on the right.

    When air starts moving, known as **dynamic,** the amount of pressure that air pushes on surfaces goes down. In fact, the higher the speed, known as **velocity**, that we go, the lower the pressure gets. This is expressed as a simplified form of the **Bernoulli Equation**:

$$\frac{1}{2}\rho\left(v_1\right)^2 + p_1 = \frac{1}{2}\rho\left(v_2\right)^2 + p_2$$

ρ	– density of the air at that point
p	– pressure at any given point
v	– velocity of the air at a given point

    The easiest way to show that pressure decreases (or pushes less) is with a simple experiment. We start with an index card (or a playing card from an old, no longer used deck) and push a thumbtack or small nail through the very center. Now place a spool of thread (can be an empty spool) on top of the tack or nail. We then pick up and hold the whole assembly together and start blowing through the hole in the top of the spool (see the figure below).

Blow in here.

Low pressure from moving air.

Full pressure from non moving air.
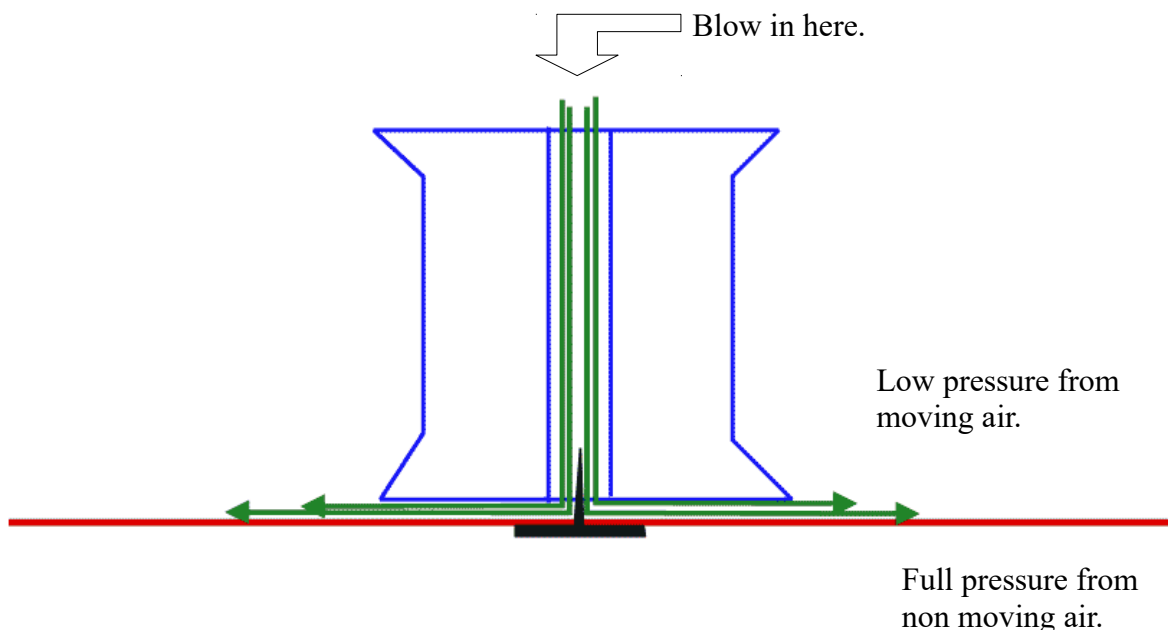
Don't let go of the card until after you have started blowing. The reason for this is that we initially must apply a resistance force to "turn" the flow parallel to the card. After the flow has turned, if we let go of the index card and just hold the spool then the card will stay floating as long as we can keep blowing. But as soon as we stop the card falls to the floor. This happens because the air starts moving parallel to the card and reduces the air pressure on the top. But at the same time there is no motion on the bottom of the card and the pressure is higher. There is no longer a balance of force on the card. In fact, there is enough unbalance to overcome the pull of **gravity** on the card.

While we are on this experiment we are going to sidetrack just a little. Any time motion starts, there are things we must do or things that happen as a result of the beginning of the motion that do not continue after the motion is established. These beginning conditions are known as ***initial conditions***. Conditions that exist once the motion has passed the beginning and do continue for the rest of the motion are called ***steady state conditions***. The effects of initial conditions can be seen on an airplane including paper ones. When we first toss our plane we can sometimes see some wiggling, or a sudden drop, or even a sudden climb, then things calm out and our plane starts flying smoothly.

The pressure pushing on the bottom side of our card where there is no motion is called ***stagnation pressure*** or ***total pressure***. The pressure that is pushing against the top surface of the card where there is motion is called the ***static pressure***. And the pressure reduction on this surface due to air movement is called ***dynamic pressure***. Let's look at how our equation would now look:

$$\frac{1}{2}\rho\left(v_1\right)^2 + p_1 = \frac{1}{2}\rho\left(v_2\right)^2 + p_2$$

$$\frac{1}{2}\rho\left(v_{dynamic}\right)^2 + p_{static} = \frac{1}{2}\rho\left(0\right)^2 + p_{total}$$

$$\frac{1}{2}\rho\left(v_{dynamic}\right)^2 + p_{static} = p_{total}$$

How Can Air Travel Faster Over My Wing Than Under My Wing?

Now we are finally going to get to talk about airplanes or at least wing sections called **airfoils**. Now, if we actually tried to draw the motion of individual air molecules, that would be impossible since there is going to be a lot of random motion; remember that third property of gases that we talked about so long ago. What we can draw is the average line of travel for our energetic air particles. With this we will start to have a meaningful diagram. These average lines of travel are called **streamlines**.

Now for a minute let's look at what streamlines would look like over an airfoil if air traveled the same speed on top as on the bottom. Note that in the figure, air traveling on the bottom would get to the back edge of the wing first due to the shorter distance it would travel. Since we can't leave a gap with no air on top of the wing the air on the bottom would have to turn the corner and come up to meet the air traveling over the top of the wing. This is what theory would predict as the motion of an **ideal gas** (not the same as a perfect gas) around an airfoil. An ideal gas is one that has no attraction between the molecules. If this happens then the speed is the same on both top and bottom making the pressure the same on both sides and leaving no net force or **lift** on our wing. So it won't fly. It's a good thing air isn't ideal.

Air Flow of an Ideal Gas
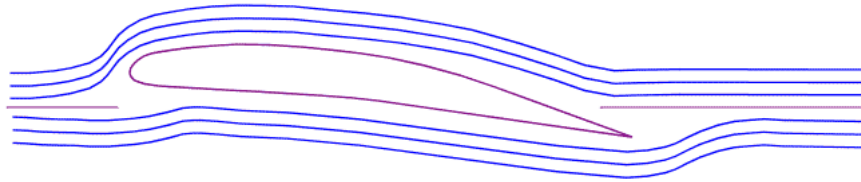
1

Remember the second important property of air where we said that air molecules were not very attracted to each other? Well "not very" is not the same as "not". The little bit of attraction we do have between molecules of air is absolutely essential to flight, be it birds, bugs, flying squirrels, kites, or airplanes. This attraction between molecules makes air somewhat thick and sticky, a property known as **viscosity**. This stickiness is not near as much as honey or tar, but is thick and sticky enough that it refuses to allow the gas to turn a sharp corner.

The proof of this is extremely complex. If we write the mathematical equations for the motion of air particles, they are so complex that we can't solve them. These equations are called the **Navier-Stokes equations** and are useful when we can make assumptions and ignore parts of the equations under certain conditions.
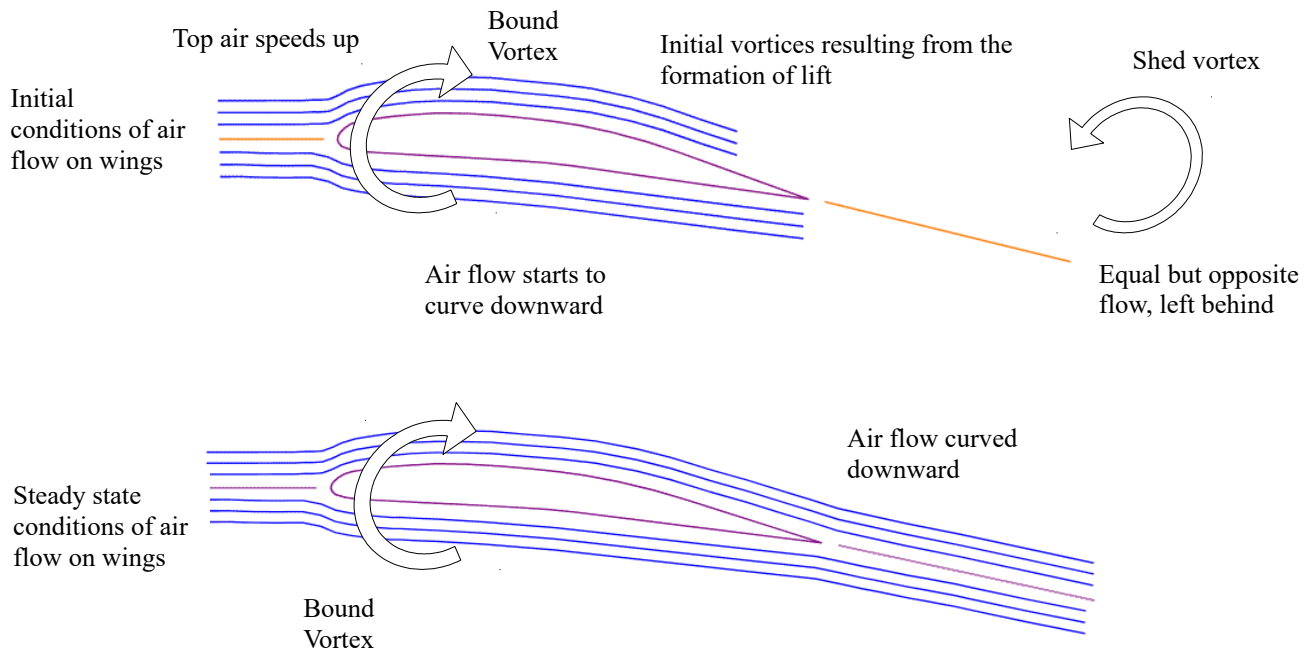
But when it comes to real flight, we can't theoretically predict what is going to happen. Since we can't solve the problem from theory, we must work from observed behavior. And when we mathematically analyze what we observe, we develop what is called **empirical** equations as opposed to **theoretical** equations, which are derived from proven laws.

The observed fact that air won't turn a sharp corner is named the **Kutta condition**. It is actually stated as follows: *A body with a sharp trailing edge in motion through a fluid creates about itself a circulation of sufficient strength to hold the rear stagnation point at the trailing edge*. This is a very long way to say that the air won't turn the corner. If we look at the geometry of the airfoil we find that the distance along the top is longer than the distance along the bottom. This will force the top air to go a bit faster than the bottom air. The streamlines, once flying at a steady speed, are going to look as follows:

Air Flow of a Real Gas
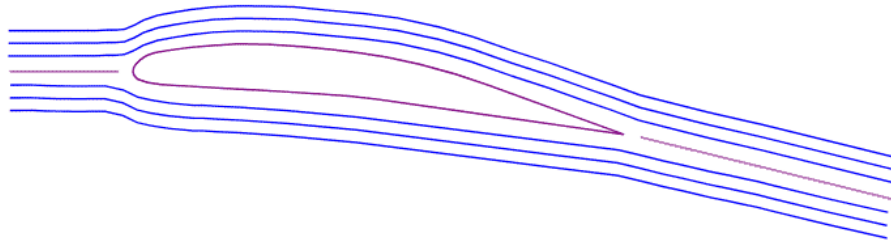
Now if air won't turn the corner what really happens then? Remember we mentioned earlier about initial conditions while discussing our spool experiment? If not, you had better go back up there because we're going to talk about them again. That's because an airfoil in real air starts to act like the ideal gas with air going the same speed on top as on bottom. But as the potential of a vacuum starts forming at the top rear of the wing due to the fact that the molecules at the top are lagging behind the ones on the bottom, there is an initial roll of the air, called a ***vortex***, at the rear of the airfoil. There is also a vortex at the front of the wing that is equal strength but opposite direction creating a ***vortex pair***. These vortices are the result of air on top of the wing speeding up and forming the downward slope of the streamlines. Aren't you glad for illustrations because words are hard to use to completely describe what's happening here? The original rear vortex is shed and left behind on the runway, and can be seen in movements of dust or smoke for larger aircraft.

Top air speeds up

Bound Vortex

Initial vortices resulting from the formation of lift

Shed vortex

Initial conditions of air flow on wings

Air flow starts to curve downward

Equal but opposite flow, left behind

Steady state conditions of air flow on wings

Air flow curved downward

Bound Vortex

The front vortex stays bound as initial condition turn to steady state to keep air flowing down the wing. Let us go a step further; reason tells us that as the plane continues traveling, the air must eventually return to stagnant somewhere behind the airplane.

Air Flow at Wing Area

Stagnant Air

Low Pressure
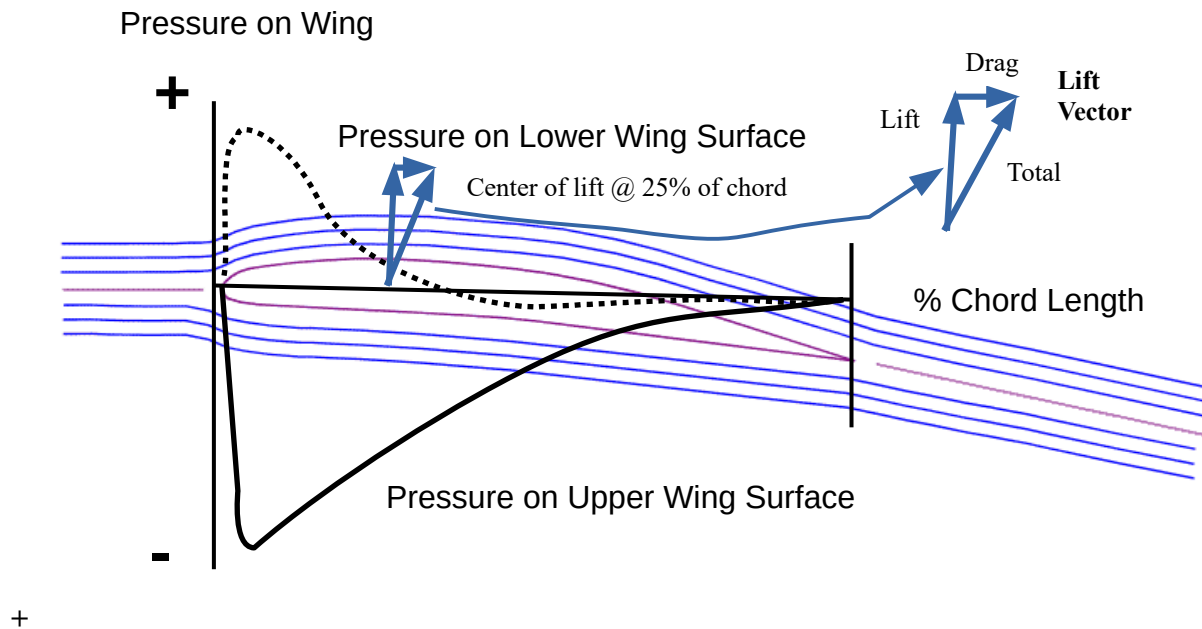
Transitional
Region

Stagnant Air
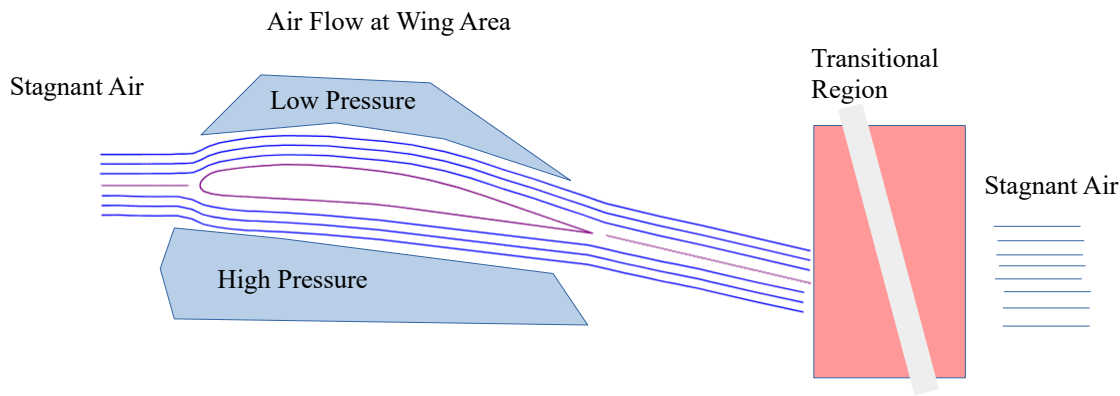
High Pressure

If a vortex pair is required to start air to curve, what happens to cause the flow to transition from curving back to stagnant? To get back to stagnant conditions, air from the high pressure area (bottom) must flow to mix with the air in the low pressure area (top). To do this, a new vortex pair is required. The new pair is located behind the wing and becomes a vortex on both left and right of the wing. Their direction is to turn airflow from under the wing to the top. The transition length varies by properties such as: velocity of the plane, chord length, length of the wing, as well as other conditions.

Right Vortex

Bound Vortex

Left Vortex
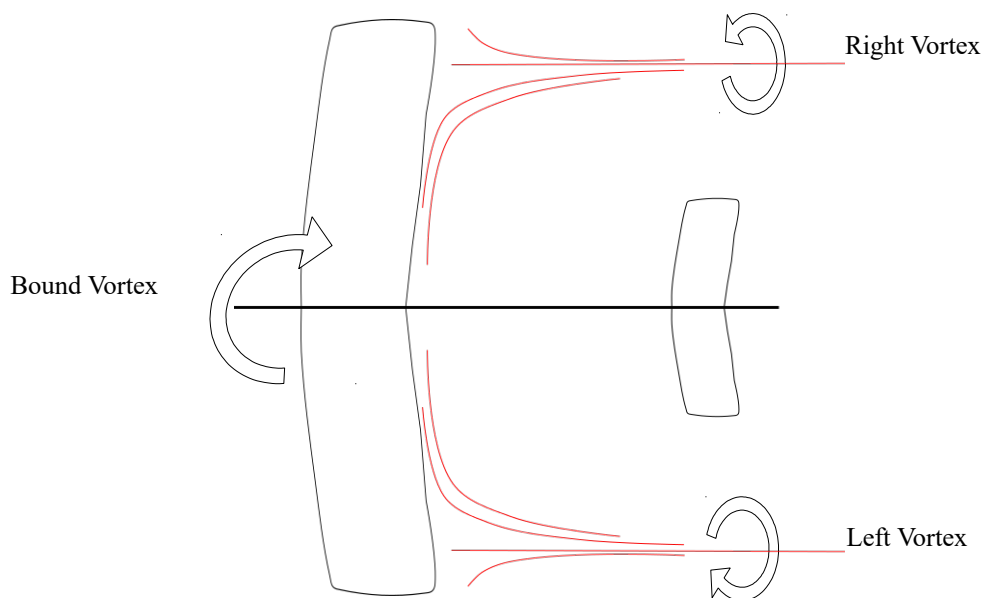
Does a vortex pair always start behind the wing as shown? No, the vortex pair can start at the leading edge of the wing, creating loss of lift on the outer edges of the wings. Worse, the vortex pair can start at different places on the left and right wings causing uncontrolled rolling. The answer to both of these conditions is to add winglets to the plane design. Winglets allow us to control side movement of air as air travels over the wing and set up a proper Left/Right vortex pair.
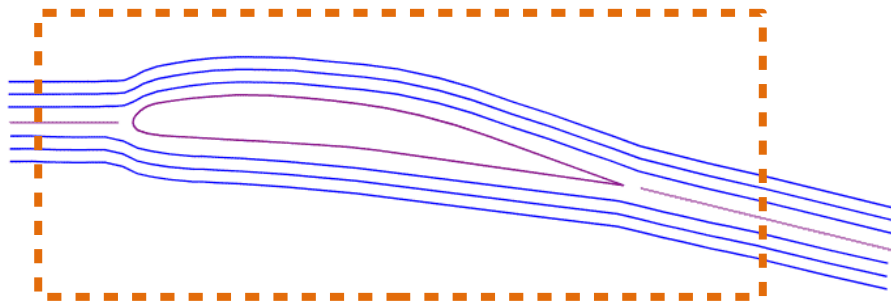
There are a few more things that we need to know about the flow around an airfoil. One of these is that the airspeed at the front edge of the wing and at the back edge of the wing, both on top and on bottom of the wing, must be the same. When we graph both top and bottom airspeed (normally graphed as the air pressure, since air pressure and airspeed are proportional) from front to back of the wing, we find a distribution similar to the one below:

Pressure on Wing

+

Pressure on Lower Wing Surface

Center of lift @ 25% of chord

Drag

Lift

Lift Vector

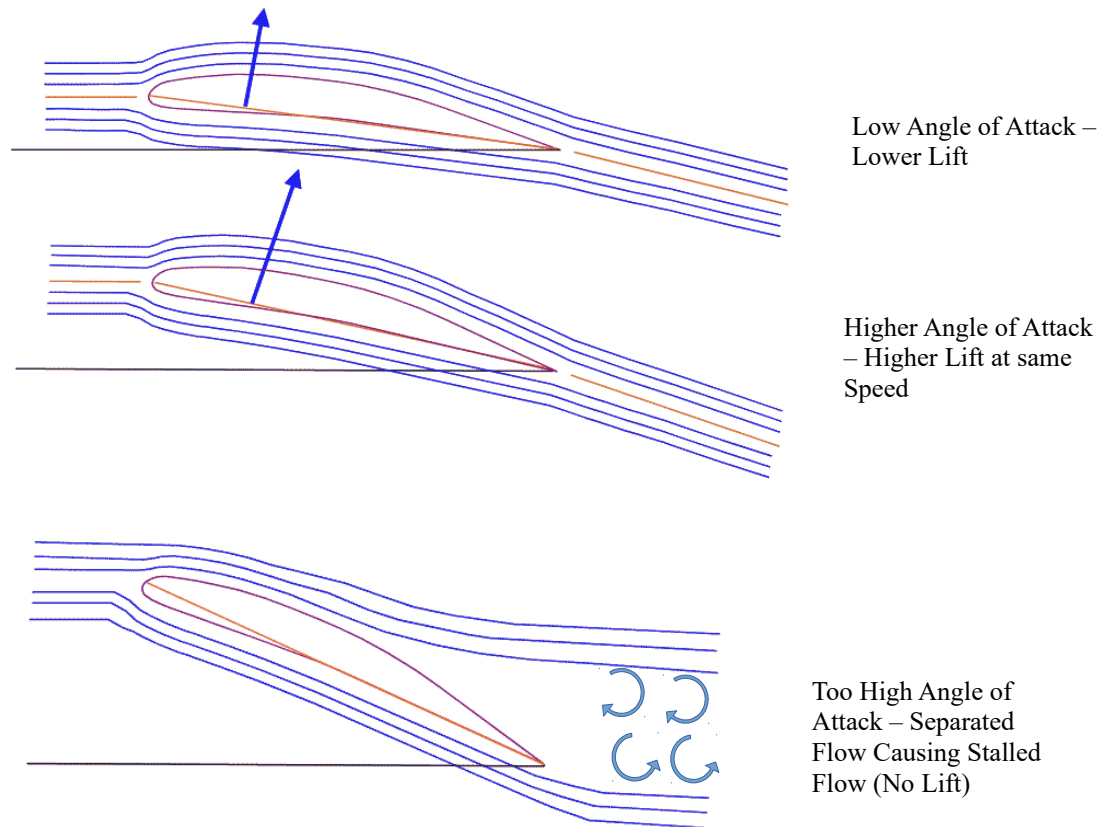Total

% Chord Length

Pressure on Upper Wing Surface

−

+

The actual shape of the pressure lines depends on many variables, including wing shape and angle of attack. When we look at the difference of the combined air pressures we can find the centroid of the area bound by the top and bottom curves. This is the point that lift acts through, and is known as The ***Center of Lift***.

Look at the diagram and you will note that the result of the "push" of the pressure is a ***vector*** (has both magnitude and direction). This vector has both vertical part (***lift***) and a horizontal part (***induced drag)***. The induced drag is added to the ***frictional drag*** of the air on the plane to get the ***total drag***. We want to note one more thing about lift. Lift does not come without adding drag. This additional drag is why darts will normally travel further than gliders.

Also, we need to know when we draw a box that completely surrounds the wing, as shown below, we find that air comes into the box horizontal and leaves angled down. By Newton's Third Law, *for every action there is an opposite and equal reaction*, if the air is pushed down then something, our plane perhaps, is pushed up. It's true the plane was pushed up and an equal force pushed down the air.

Next consider lift changes as the wing rotates in the pitch axis:

Low Angle of Attack –
Lower Lift

Higher Angle of Attack
– Higher Lift at same
Speed

Too High Angle of
Attack – Separated
Flow Causing Stalled
Flow (No Lift)

We can note that the lower figure has more *angle of attack* and that this results in an air flow where the air coming off the trailing edge has a more downward direction. From Newton's Law we can note that a larger action requires a larger reaction or in other words more lift. However, we cannot keep increasing the angle of attack to generate more lift. The reason for this is that we finally reach a point where the air will not follow the surface of the wing from leading edge to trailing edge. The air flow becomes unattached and we lose all lift. This condition is known as ***stalling***.

Next thing to know is that the amount of lift is also ***proportional*** to the average speed of the air over the wing (speed the plane is moving). Being proportional means that as speed goes up so does the lift. We maintain the lift we need to match the weight of the plane as we slow down increasing the angle of attack. From the above discussion about stall we can see as we slow down the angle of attack must increase until we reach the angle where we will stall. This is our minimum flight speed.

How much lift we get and this minimum flight speed vary from wing to wing, which is why our paper planes work at different throwing speeds. There are many variables that will have an effect on these values. These variables include the wing geometry and the texture of the paper. We will get a little more in depth on this subject in our next section.

Now we can get back to our question for this section: How can the air travel faster over the top of my wing than under the bottom? Maybe you caught the answer, which is to give the wing a sharp edge at the rear. So we have finally answered the question of how an airplane flies. It should seem that we could stop with the topic of aerodynamics. But there are a few extra topics to cover to be more complete. First someone might just ask:

My wings are flat so why would they act like an airfoil?

The Kutta condition is actually going to make air flow over a flat plate act very much like it would flow over an airfoil. The truth is that an airfoil is simply a modified flat plate that is designed increase lift and to reduce frictional drag over the wing. To understand how air travels further over the top of a flat plate than on the bottom requires us to bring up another topic. This is definitely one of the most important topics in aerodynamics – **boundary layers**.

In the last section we mentioned that wing geometry and paper texture both had an effect on the lift, drag, and minimum flight speed. How these variables affect the lift requires the analysis of what is happening at the air that is very close to the surface of the wing where all the friction between wing and air is occurring. This is the boundary layer and its study is too complex to completely cover here. But we are going to look at some of the characteristics of the boundary layer.

We need to make one quick point here, and that is that there are two types of incompressible flow: laminar and turbulent. **Laminar flow** is a very smooth flow where the air travels along streamline with very little deviation. **Turbulent flow** is very "choppy" and does not consistently follow a steady streamline. As an example think of a sheet hanging on a clothes line; a slow breeze will cause it to rise smoothly but a heavy wind makes it flap violently. This is the difference between laminar and turbulent. Both conditions will produce lift and allow flight but laminar flow produces much less friction and therefore has less drag. We can predict when flow will change from laminar to turbulent by looking at the **Reynolds number**. The Reynolds number for any given wing flow is calculated as follows:

$$R = vl\rho/\mu$$

R – Reynolds number at a point
v – velocity of the air at that point
l – chord length of the wing
ρ – density of the air
μ – viscosity of the air

For a given surface geometry and texture, there is a Reynolds number that will cause flow to change from laminar to turbulent. With the low speeds of paper airplanes our Reynolds number will stay low and the flow remain laminar. This is the only condition we are going to study.

Getting back to our study of boundary layers, we need to know exactly what a boundary layer is and what causes it. We are going back to our property of viscosity, that little bit of stickiness air molecules have for each other. The air molecules also have stickiness to our paper. This causes the air molecules next to the surface of our paper wing to stick to the wing so that they have almost no velocity relative to the wing. Just above these are molecules that have just a tiny bit more speed than the first layer. One more layer up and we are moving just a little faster. Note: there is loss of energy due to decrease of velocity as we move from top to bottom of the boundary layer; this is frictional drag. Eventually, we will get to molecules that are moving full speed of the flow relative to the wing. The air between the wing surface and this first layer of full speed air is called the boundary layer. It can vary in thickness depending on many factors including the geometry, surface texture, and Reynolds number as mentioned above. As it varies in thickness it changes the effective or **virtual shape**. We can visualize the speed of each location in the boundary layer as shown:


Full speed zone

Boundary layer zone
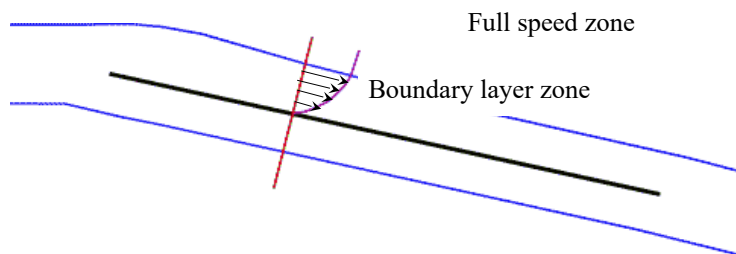
From here we want to look at the shape formed when we plot the end of the boundary layer at each cross section. Remember the thickness of the boundary layer changes as we travel down the length of the wing. In this figure the boundary layer is shown in green. What we can start to see is that the outer limit of the boundary layer takes on the shape of an airfoil. The streamlines act as if they are moving around a solid that has the shape of the boundary layer. This is the ***virtual airfoil***.
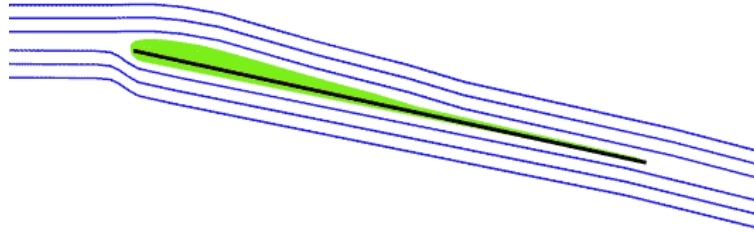
An interesting point is that the pressure on the wing is the same value as the static pressure in the full speed flow (speed away from the boundary layer). This happens because even in compressible flow, complex changes of the fluid properties occur in the boundary layer. These property changes include temperature, pressure, and of corse velocity. The temperature effect here is due to friction, which converts kinetic energy to temperature increase, which is yes more drag; this is the frictional drag mentioned before.

When our flat plate is made of flexible paper it will bend under pressure to produce a virtual airfoil more like this one.
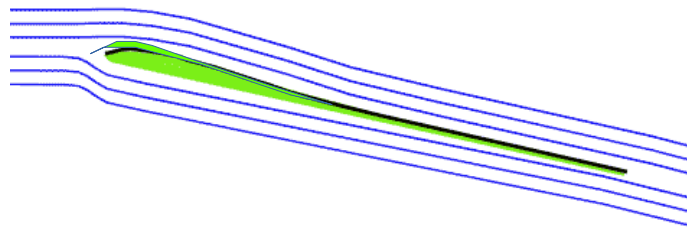
If we use NASA's FoilSim III software (full version) we find that the lift to drag of a flat plate is 11.28 (for C_l =.5) but if the curve (called ***camber***) of the paper is just 1.0 % of the length then lift to drag goes up to 12.005 for the same lift (Angle of Attack drops from 4.76 deg. to 3.6 deg). The thicker boundary layer that is formed near the leading edge causes extra lift but we still have a lot of friction drag, 6.82% lower then a flat sheet, according to FoilSim III. We can improve our flight time even more by lowering this drag by folding the paper so that we "fill in" some of the virtual shape as shown in the next figure. Enough folds to get about a 3.0% camber and 3.0% thickness (Flat Bottom shape) shows angle of attack required to drop to 1.28 for the same lift and reducing drag another 4.7%.
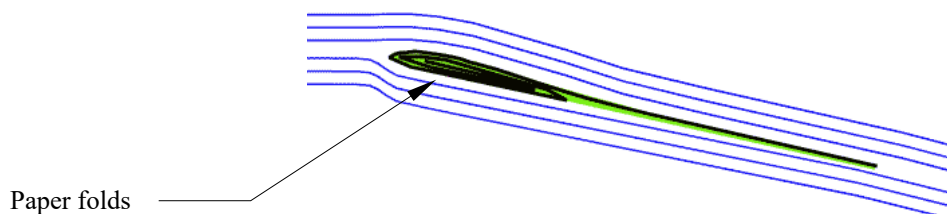
Paper folds

There are conditions (mainly very slow speed and sharp corners) where the paper folds cause some air to get "trapped" and the boundary layer is between this trapped air and the full flow. This trapped air further makes our virtual wing more like the flat bottom model in FoilSim.

Trapped air
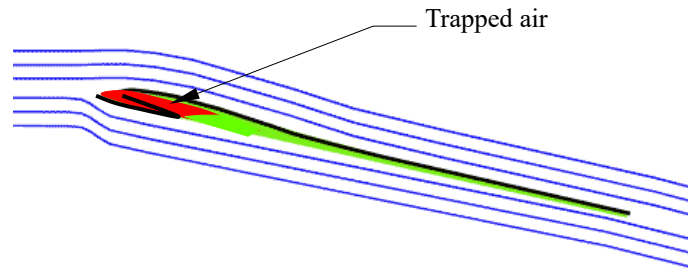
The length of the virtual airfoil that is efficiently producing lift does not exist the whole length of the wing. This basically means that only the front section of the wing is producing the majority of the lift. Back in the 1960's it was a popular idea to cut out the low lift area to reduce weight and drag. I don't know how much improvement in flight was made, but the planes did look more like full size planes. However, this idea is worth experimenting with.

Lower lift area can be cut out.

Effective wing
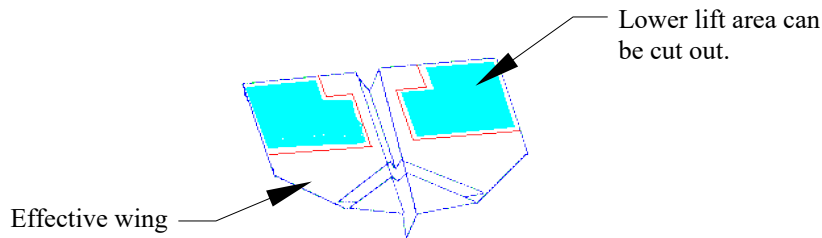
Before we completely finish with boundary layers, we want to consider why this layer is so critical in determining how much frictional drag is produced as air moves across our wing. To start we need to understand about shear. There are three ways that we can apply equal but opposing forces into a body. The forces can squeeze, squash, or warp. These actions are called compression, tension, and shear. These can be illustrated as follows:

Types of Loading

Compression   Tension   Shear
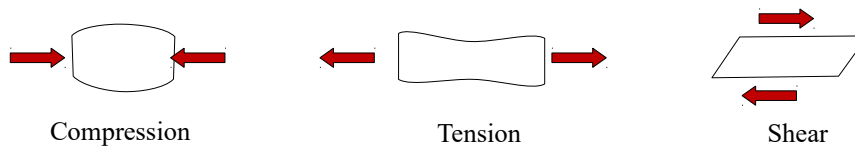
Within the boundary layer, each individual layer of air is being sheared between the layer below and the layer above. This shearing produces the different speeds between layers as individual air particles slip across each other. As the particles slip over each other there is an attraction between them to resist this slippage. As the particles overcome this resistance they lose dynamic energy in the direction of flow. This loss of energy is in the form of a force, friction, acting through the length of the boundary layer. A student of physics will recognize that any force acting through a distance is work. And that work can change energy of motion (kinetic energy) into heat energy.

Why Do Some of My Paper Airplanes Fly Poorly?

One of the worst things we can do is have a paper that is not very flexible. As we were talking about boundary layers above, we probably should look at what is happening to the lift and drag. A very important characteristic of a paper airplane is the lift to drag ratio. Since we only have our initial toss to get us as far as possible, it becomes very important that we have a good lift to drag ratio (a ratio is the value we get by taking the first number and dividing it by the second number). In terms of boundary layers, each of the following steps is an improvement. Our paper:

1. Acts like a flat sheet.
2. Curves to shape of the boundary layer.
3. Fills in the gaps in the paper folds to improve the shape (real and virtual) of the airfoil.

To get from step 2 to step 3, we design the number and geometry of the folds. But to get past step 1, we must have a good flexible paper to make a good paper airplane. If our paper is stiff, we lose some of our lift to drag ratio and our paper airplane will not fly as far. If it is too stiff it may curve more on one side than the other; our wings become unbalanced and the plane will roll or even go into a spin and not fly at all.

Next we are going to look at one more effect sharp corners can have on our paper airplane. If you read the section on trimming, you may have noticed that it was mentioned that if you did not use tape then you may have to smooth out the edges between the plane's body and the wings. The reason for this is that if the plane has a tail then the tail will act like a plow and force the air coming down the body channel to flow up and over the wing as shown below. If there is a sharp corner between the body and the wing then the air will not turn the corner smoothly. If the air does not turn smoothly, it will cause poor lift over much of the wing and the plane will fall. This is the only reason to use less "crisp" folds.

View of plane looking from nose to tail

Sharp corner
No turn of flow

Rounded corner
Smooth flow
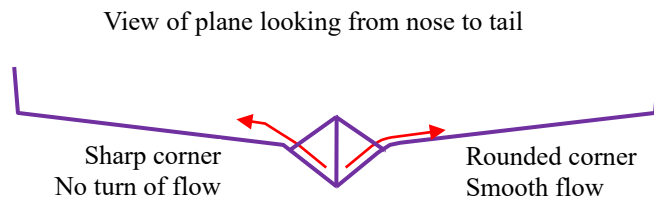
It does not take much of a roundness to allow smooth flow. This can also be a real problem if one edge is rounded enough for smooth flow but the opposite edge is not. This causes unbalanced lift and the plane will begin to roll, making the plane unstable. [Note for Thought: If rounded corners allow smooth flow and sharp corners are needed at the trailing edge of a wing for lift then what happens if icy buildup occurs on the trailing edge of a wing? Lift is drastically reduced as air flow can turn the rounded corner. In fact, ice changes the whole shape of the wing (resulting in reduced lift and increased drag) and its texture (that can cause changes in the boundary layer resulting in wing turbulence, unattached flow, and stall). Especially a problem if one wing differs from the other! Small aircraft without deicing systems need to avoid icy conditions!] This rounded vs smooth edge instability brings up another interesting question:

The next cause of poor flying is instability. If a small change being made to the **orientation** of our airplane causes a larger change to occur, then we have an **unstable** condition. However, if a small change in the orientation causes the plane to restore itself or at least to resist additional change to the original orientation, then we have a **stable** condition. To understand what we are

referring to here we must know what orientation means. Orientation is the direction our plane is pointing and a change to the orientation requires a rotation.

This leads us to look at the axis system we use to define our plane. This system is called the body-axis system and has six degrees of freedom as shown below:
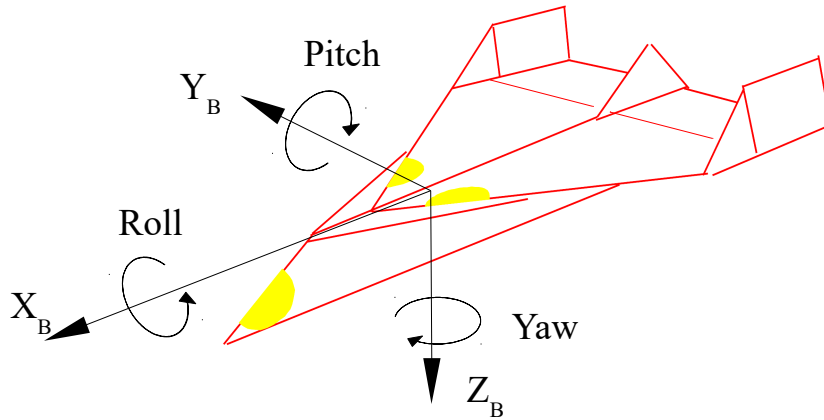
The instability caused by not smoothing the folds is in the Roll direction. Another cause of roll instability is if we need winglets on our wings to keep air from flowing off our wing sideways as shown below. The amount of lost lift due to this action is dependent on the ratio of wing span to the chord length known as the ***aspect ratio***. A plane, such as a paper airplane, with shorter wings can lose a significant percentage of lift from this effect. It is prevented by adding winglets. However, if the aspect ratio is significantly higher, there is less chance of this flow having as much effect. Elliptical wing shapes have less issue with this instability.

Bend down or up 90°
to stop this flow
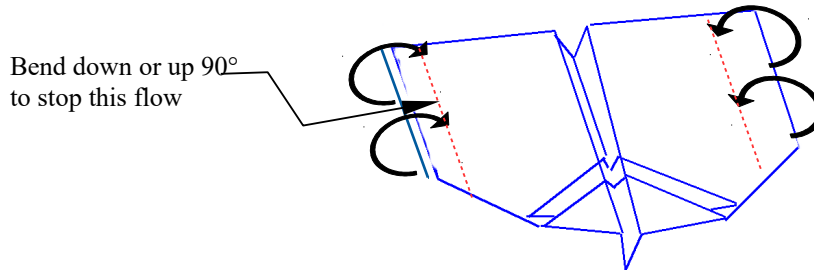
There is one more condition that can cause roll instability and has happened in badly designed full size aircraft. This one is not easy to follow but can be deadly. It is instability caused by a negative ***dihedral angle*** (the angle the wing makes from horizontal).

Positive dihedral
Stable condition

Negative dihedral
Unstable condition
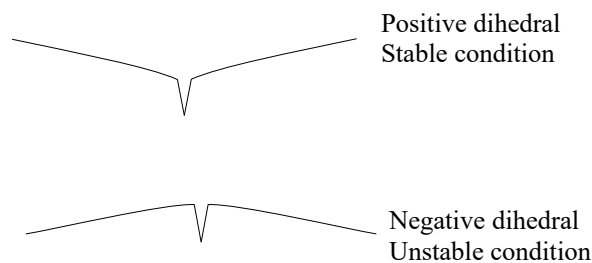
To visualize this type of instability, make two basic box planes. For one plane fold the wings with a positive dihedral and the other with a negative dihedral. Now start with the positive one. Hold it in

front of you and pointed away. Move the tail to the left and nose to the right, which is a slight rotation in the yaw axis. This can happen from a gust of wind hitting the tail. Just remember that the plane is still traveling away from you and no longer traveling the direction it points. If you look very carefully, you will notice that this makes the air come from a lower angle to the left wing increasing the wing's angle of attack and the lift on the left wing. This extra lift causes the plane to roll, making the left wing go up. That in turn increases the right wing angle of attack. The wings balance their lift, the plane banks to the right and will level off. We are now headed a new direction but no danger occurred. This is a stable reaction.

Now try the same thing with the negative dihedral wing. Move the tail to the left and the nose to the right. The air now flows more from the top of the left wing, lowering the angle of attack and lift on the left wing. This will cause the plane to roll left wing down, right wing up. This in turn makes the angle of attack even worse for the left wing and the problem just gets bigger. Very quickly the plane will be rolling counterclockwise as seen from the rear. Lift will no longer occur on either wing and the plane will spin out of control towards the ground. This instability is known as the Dutchman's roll and has killed many pilots.

Instability in the pitch axis is caused by the lift not being in line with the paper airplane's ***center of gravity***, or balance point. This will be discussed in a later section on pitching moments. For a paper plane, instability in the yaw direction is a design flaw that causes the wing to stall (normally one wing first). This usually occurs when the plane is moving too slowly to produce enough lift to keep our plane in the air. This causes a motion of both a roll and a spin in the yaw direction initially, but as the wing stalls even more, the yaw worsens, leaving the plane falling to the ground in what is known as a flat spin. Often it is caused by the center of gravity being too far back and making the nose pitch too much upward.

What Other Equations Should I Know About Airplane Dynamics?

Remember the sketch below from our earlier discussion of airplane loads? A few of the laws of physics that would help the analysis of the effect of these loads on our plane are: ***Newton's Laws***, ***Conservation of Energy***, and ***Conservation of Momentum***.
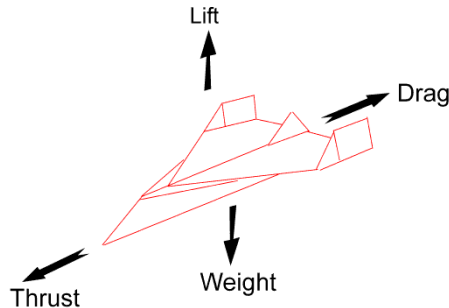
We will start with Sir Isaac Newton's three laws of motion and Law of Gravity:

First Law – If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).
Newton's First Law basically states that we must apply a force to start our airplane flying. This may seem quite obvious at first, but it also states that any change in motion requires a force. This implies that since we can observe our paper airplane slow down, then we can conclude a force is working on it; specifically we call this force drag. This law also shows the importance of direction, implying that force and straight line speed (velocity) are vector quantities.

Second Law – If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant force and in the direction of this resultant force.
The second law is an extension of the first law and gives us the basics for generating an actual equation comparing the amount of force to the rate of change of the straight line speed (acceleration). It adds the concept of a constant of proportionality as a property of the particle. This constant of proportionality is known as mass. We can measure it by finding the amount of force required to balance it under the influence of gravity. This balancing force is known as weight. This law can be written by the equation:

$$F = ma$$

F – force on the particle
m – mass of the particle
a – acceleration of the particle

Third Law – The forces of action and reaction between bodies in contact have the same magnitude, same line of action and opposite sense.
The main ideas we will pick up here in terms of paper airplanes is that in order for air to "push" a paper airplane up (lift), then the paper airplane must equally push the air down. The section on airflow over a wing demonstrates this concept. Another use for this law, in understanding full size airplanes, is that to maintain speed we must push on the air the same as the air pushes back. This is why an engine can push against the air in order to generate thrust.
 Newton's Law of Gravitation – Two particles of mass M and m are mutually attracted with equal and opposite forces F and –F of magnitude F given by the formula:

$$F = G\frac{Mm}{r^2}$$

F – force of attraction on each particle
M – mass of the larger particle
m – mass of the smaller particle
r – distance between particles
G – gravitational constant

Newton's Law of Gravitation tells us that gravity is pulling our airplane down. It allows us to calculate how much lift we need to generate in order to oppose gravity. We often talk about the acceleration of gravity. If we take the above equation and substitute mass (of our airplane – "little" m) times acceleration for force and then divide both sides by "little" m, we get the following:

$$a = G\frac{M}{r^2}$$

If we are anywhere near the Earth's surface, the value of r² is constant as is the mass of the Earth, M, and G, the gravitational constant. This means that acceleration is constant and can be calculated to be 32.17 ft/sec².

The next topic we need to understand is about moments and rotation. Above we talked about forces that were in a straight line of action with each other. But what happens when the forces are not in a straight line? The answer is that we find that there is an equivalent set of laws of motion that deal with a body as opposed to a particle. The difference between a particle and a body is that a body has size where a particle does not. A particle is only a theoretical object since all solids in the real world have size.

Any misalignment between forces is called a moment, and a moment will cause a spin that is equivalent to speed, called angular velocity (represented by the symbol ω), to change in value. The rate of this change is called angular acceleration. The equation for this relationship is:

$$M = \bar{I}\alpha$$

M – moment applied to the body
$\bar{I}$ – moment of inertia of the body (resistance to change in spin)
α – angular acceleration

For our purposes we are not going to concentrate on this equation too hard. What we do want to understand is that if our forces are not lined up, this will cause our paper airplane to spin, and that this is due to the moment that is acting on the body. We do want to learn how to calculate the amount of moment a body sees. Remember the illustration below? We are going to learn how to calculate the value of the moment that would produce Pitch.
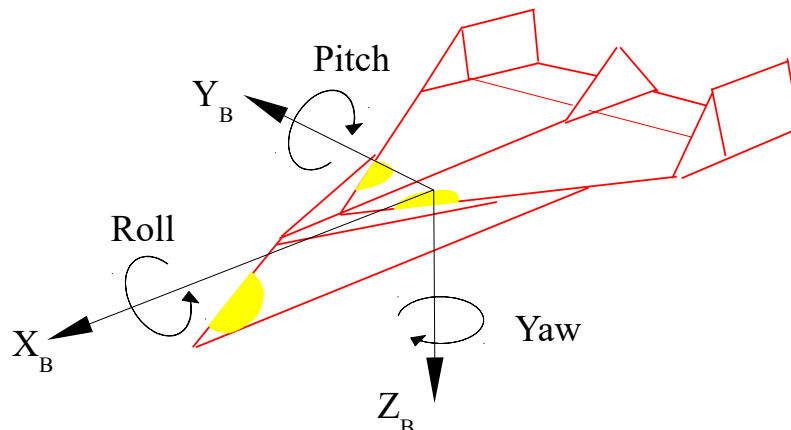
We are going to diagram the forces that act in the X – Z plane; this means the two dimensional space that would include both the X axis and the Z axis. We are also going to make the assumptions that the thrust is zero (since our body is a flying paper airplane) and that the drag acts in line with the **Center of Gravity** (or CG). Before we go any further we must define CG. It was mentioned above that a real body differed from a particle since it had size. It can also be noted that all bodies have a balancing point where we can apply a force without causing any spin. This is the point known as the CG and happens to be the same point that the body's weight will act through. All of this said, we can diagram our forces that we have left on our paper airplane:

Balanced	Nose Heavy	Tail Heavy

CG - ⊗

Lift (L)

Weight (W)

M (+)   d   M (-)   d
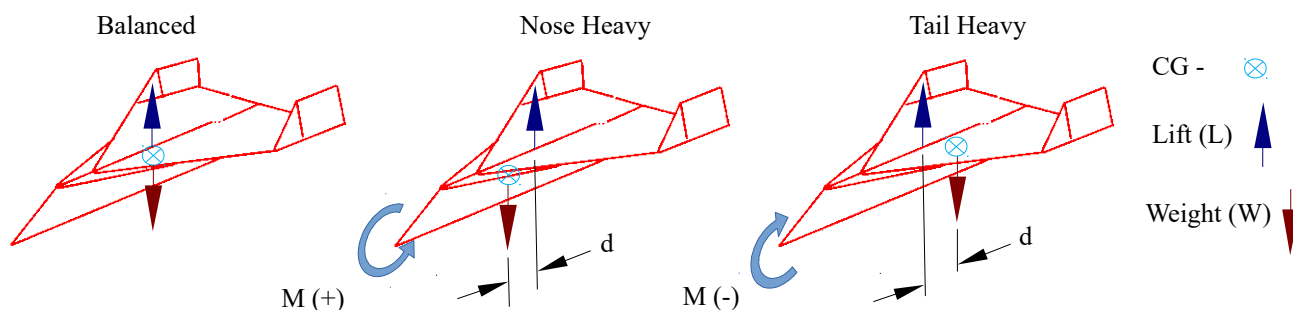
To calculate the moment (M) we need to know the value of the forces and their distances (d) from the CG. There are two sets in our current calculation: weight (W) at 0 distance and lift (L) at distance d. If the force would turn the body clockwise around the CG, it is negative, and if it would turn the body counterclockwise, it is positive (this is called the right hand rule). To get the value of the moment, we add all of the sums of the products of the forces (F) and distances (d). Using the symbol Σ to mean sum of all, then our equation for moment becomes:

$$M = \sum Fd$$

Note that for the above examples the moment is equal to the lift times the distance it is off from the CG since the weight always acts through the CG, and produces zero moment. If we analyze the trimming of a paper airplane, we will see more forces involved since we will add the effects of a tail or of a trimming adjustment that acts like a tail. We can now see how in figures 2 and 3 we must add an additional force to get a moment that will cancel out the moment produced by the misalignment of the weight and the lift. This following example will show how folding a tail (creating a tail force - F_t) balances a nose heavy paper airplane.

F_t M (-) d_2

L   W d_1

M (+)
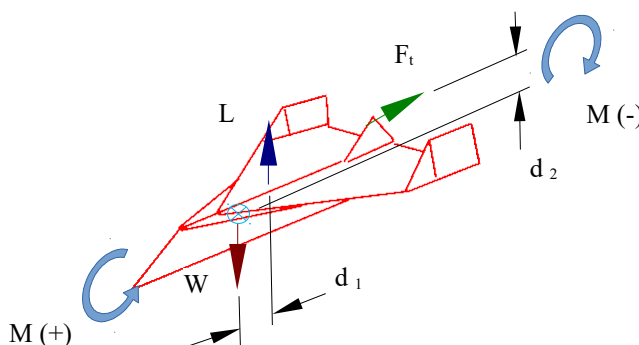
Remember we said we would discuss pitch stability later? This is the place to discuss it. If we consider a plane that is tail heavy and has an upward air draft on the tail, then the angle of attack goes down; we lose lift, making the angle of attack go further down. The same plane with a down draft on the tail would cause increase of angle of attack, which increases lift and causes a further

increase of angle of attack. This is unstable. While a nose heavy plane experiencing an upward draft would also have a decrease in lift, this would bring the nose up, restoring lift. Also, a down draft causes angle of attack to go up, causing lift to go up, thus, lowering the nose and restoring lift back to its previous level. These balancing effects make the nose heavy plane more stable. This is how most paper airplanes maintain stability, but most real planes (and even card stock planes) have horizontal tails that produce a negative lift (downward lift) to produce the same negative moment.
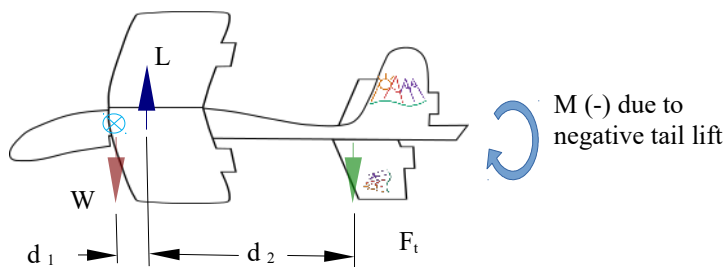
We can also analyze the dynamics of our flight by using either work/energy or impulse/momentum methods. To do this we must learn a little more Physics. If we have a force which pulls on a body as that body moves a distant d, then this force does work on the body. The value of this work (W) is the product of the force (F) and the distance (D) it pulled the body through. This work will cause a change in the energy due to motion or **kinetic energy** (E_k) of the body, which is equal to 1/2 of the mass (m) times the speed (v) squared. Let's take a look at this in the form of equations:

$$W = Fd \qquad \text{and} \qquad E_k = \frac{1}{2}\left(m v^2\right)$$

Using the symbol Δ to mean "change of", or the amount of change in, then the laws of Physics tell us that the following equation is true.

$$\text{(Work)} \qquad Ft = \Delta mv \qquad \text{(Kinetic Energy)}$$

This relationship is known as the work-energy relationship. It is interesting to note that if we push very hard on a wall that does not move, then we are not doing any work! Our body **is exerting energy** in the muscles, but it only converts **to heat energy** in our body.

The next relationship we need to know is the impulse-momentum relationship. Impulse (I) is defined as the product of a force (F), acting on a body, and the amount of time (t) it acts on the body. Momentum (M) is defined as the product of a body's mass (m) and its speed of motion (v). Physics once again tells us that:

$$I = Ft \qquad \text{and} \qquad M = mv$$

$$\text{Then}$$

$$\text{(Impulse)} \qquad Ft = \Delta mv \qquad \text{(Momentum)}$$

Examples of how these can be used are:

Max time possible for flight with force of Drag: $(Drag)t = \Delta mv$ or $t_{max} = \dfrac{\Delta mv}{(Drag)}$

Max distance possible with force of Drag: $(Drag)d = \dfrac{1}{2} m\Delta v^2$ or $d_{max} = \dfrac{\frac{1}{2}m\Delta v^2}{(Drag)}$

Note that these are the absolute maximums and the actual values will be less for a couple of reasons. First, as the airplane starts to slow down, the angle of attack must increase to maintain lift (another good reason to have CG of the plane in front of lift). This increases drag even more (you can verify with FoilSim III software). So the plane slows even faster. Also, we can't maintain flight all the way to zero speed because our wing will stall first, ending our flight. It might be fun to see just what percentage of this max we can actually achieve by experimenting.

The above equations apply when loads (through time or distance) are applied to solid bodies. But air like all gases is made of many, many particles. To apply these same concepts to systems of particles requires a whole new set of equations. These equations are called the ***Navier-Stokes equations*** which we mention earlier when discussing gasses. Even though the full equations can't be theoretically solved does not mean they don't have value. And although we aren't going to go into any details on how these equations are simplified, it is always nice to be able to display them on any presentation board for a science project on flight. They can be shown in several forms but here is one of the most common:

Continuity of Mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left[\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Continuity of Momentum

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}\right]$$

Continuity of Energy

$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(\wp)}{\partial z} - \frac{1}{RePr}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$

$$+ \frac{1}{Re}\left[\frac{\partial}{\partial x}\left(u\sigma_{xx} + v\tau_{xy} + w\tau_{xz}\right) + \frac{\partial}{\partial y}\left(u\tau_{xy} + v\sigma_{yy} + w\tau_{yz}\right) + \frac{\partial}{\partial z}\left(u\tau_{xz} + v\tau_{yz} + w\sigma_{zz}\right)\right]$$

List of Symbols:

x,y,z - Coordinate Location in the Fluid   t - Time   p - Pressure   q - Heat Gradient (Flux)

u,v,w - Velocity Components ρ - Density E_T - Total Energy μ - Dynamic Viscosity

$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ - Normal Directional Stresses $\tau_{xy}, \tau_{xz}, \tau_{yz}$ - Shearing Stresses

Re   Reynolds Number $\left(Re = \dfrac{\rho\, VL}{\mu}\right)$ where V -Velocity and L - Chord Length

Pr   Prandtl Number $\left(Pr = \dfrac{c_P \mu}{k}\right)$ where c_P - Specific Heat and k - Thermal Conductivity

Note: Total Energy must include Kinetic Energy (E_k), Thermal Energy (E_t), and Gravitational Potential Energy(E_p).

$$E_k = \frac{1}{2} m V^2 \qquad E_t = \frac{3}{2} N k_B T \qquad E_p = -\frac{GMm}{r}$$

A close look at the above **_continuity (or "conservation of") equations_** reveals there are actually 6 unknowns and only 5 equations. And by a close look, I mean this is not automatically obvious. First, you need to know the stress terms (both normal and shear) are actually dependents of the velocity components – specifically they are the second derivative of the velocity components. Also, the actual independent variable in the heat gradient terms is the temperature, T. All this factored in leaves 6 independent variables: 3 velocity components (u, v, w), pressure (p), density (ρ), and temperature (T). Since there are only 5 continuity equations, then one more equation is required. This last equation, which must relate how pressure, density, and temperature relate to each other is called an **_equation of state_**. The very first equation we discussed in this study was the Perfect Gas Law, which is an equation of state:

$$p = \rho RT$$

> p – pressure at any given point
> ρ – density of the air at that point
> T – temperature of the air at that point
> R – the gas constant

To fit our need, one of the above variables would have to be substituted with its relationship with the other two into the continuity equations. Then the last step would be to apply all the boundary conditions—initial conditions, geometry constraints, and final conditions.

As I stated before, the full equations, with no assumptions to help simplify them, can't be solved even with today's computers. However, with just a few assumptions they can be used to allow computer models to be made and analyzed that are very close to actual measured values. This allows up front design of actual aircraft with minimal redesign required after testing.

I need to point out just a few more ideas before closing. It was discussed that lift occurs because the pressure on the bottom of a wing is higher than the pressure on the top of the wing. This would lead to the idea that a computer model should try to find the pressure at any point on the wing and allow us to calculate lift from there. In fact NASA's FoilSim III does this. But, if this is the only way we analyze, then we will only find the lift and the induced drag, not the total drag including friction. So we need to look at more than pressure. What might not be so obvious is that trying to analyze

what is happening to our wing may not be as easy as analyzing what is happening to the air – remember that energy loss we discussed when we were looking at boundary layers?
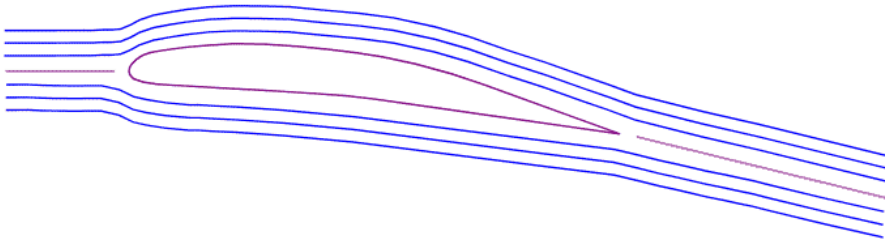
Earlier we had an illustration similar to the one above. Now suppose we concentrate on making a computer model that reflects what is happening to the air as illustrated here instead of what is happening to the wing. Our model would have to include all the air flow going into and out of the "box". Instead of concentrating on just pressure, it would look at velocity, density, and temperature of the air. This type of analysis is called ***computational fluid analysis*** and is a very valuable tool. Just remember that whatever happens to the fluid, the opposite happens to the airplane – Newton's Third Law.

In this example we look for three things. First, the downward change of direction of the air which matches the lift received by our plane. Second, we look for the reduced relative speed of the air to the wing which is from a combination of the induced drag and part of the frictional drag. Last, we look for temperature increase of the air that is a result of frictional drag. After this we could then look at the pressure distribution on the wing and know just about everything we need to analyze our wings.

Just remember no matter how good our model, we still need to test physical models in a wind tunnel to verify all is as predicted. And even then there can be surprises found in the first test flights. But today's modeling and flight analysis computer programs have made a huge impact in accurately predicting how a design will fly.

# Tips and equations for design of card stock planes

It seems like we have covered a lot; but, There is still a lot to cover as far as geometry and wing style before being able to design card stock airplanes. First lets look at basic shapes:

Rectangular Wing – The simplest wing shape and not a bad one except for one issue; the area (indicated in blue) on both wing or worse only one, can lose lift due to air flow from bottom of the wing to the top. The fix, add winglets.

Elliptical Wing – A common fix to rectangular wing lift issue with no winglets required.
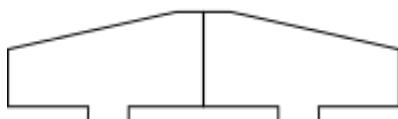
Leading Edge Taper Wing – As airplane speeds and payload weights increase wing loads also increase. This creates very high bending loads on the wings. Need for maneuverability also further increases these loads. One solution for reducing these loads is to taper the wing so that more of the lift is produced closer to the fuselage, reducing the bending loads. One of the primary types of aircraft with these high speeds and maneuverability requirements are fighter airplanes. These planes also require the best visibility possible for the pilot; there, the leading edge was usually the tapered edge.
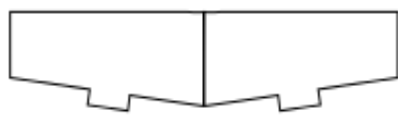
Trailing Edge Taper Wing – Rarer than leading edge taper, the primary reason to use this type of wing is when taper is required, but the plane has a more forward CG, requiring center of lift to be more forward.
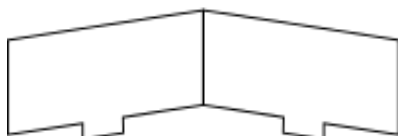
Swept Back Wing – Originally developed to improve flight stability due to turbulence, especially during maneuvering, swept back wings were developed. But it was soon found that great improvement in flight for even straight and level conditions of near sonic speeds (today's passenger jets, for example) could be achieved when localized shocks could form and cause lift issues.
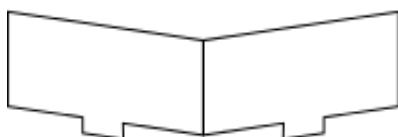
Swept Forward Wing – Primarily reserved for canard aircraft (tail in front, wing in rear) for stability and balance.
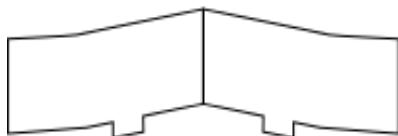
Mixed Configuration Wing – The possibilities for wing shapes become numerous if you start combining the above basic shapes.
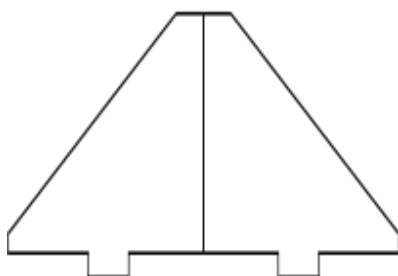
Delta Wing – A design for super sonic flight conditions where the pressure distribution on wings is not due to standard incompressible theory but to pressure differences across shock waves. However, for light weight paper aircraft this shape flies fine;

after all, most folded designs are shaped very similar. Check the section on virtual airfoils to see how sections of the wing produce very little lift in subsonic conditions. Although not shown here, super sonic craft often have wings which are not *fixed wing* style, but have ***variable wing*** shape allowing good flying conditions for subsonic, ***transonic*** (speeds high enough to produce sonic conditions at some spots on the plane, but still below the speed of sound), and super sonic flight. Doing some outside internet research will give more detail about planes that use this wing configuration.
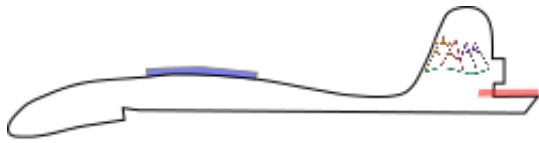
High Wing Mount – This design gives the pilot the best view of the ground. Can be of great advantage to new pilots flying purely by sight.

Low Wing Mount – This design gives the pilot the best view of the sky above. When pilots have the experience and training to fly by instruments, this can be to their advantage. Fighter pilots and those that fly using high traffic airports are two examples.
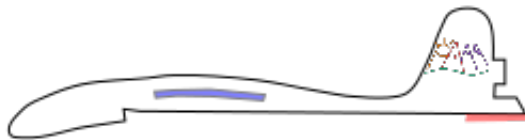
Cantilever Wing Mount – For aircraft with high wing loads (from high payload or high speed maneuverability), these wings can give a much stronger attachment to the fuselage.

Canard Wing Mount (high, low, or cantilever) – A horizontal change with the wing mounted in the rear. This makes a very stable plane.

Tail Mounting – Just like the wing, the horizontal stabilizer (shown in pink) can be mounted high, low, cantilever, or move from rear to front. Note that the vertical stabilizers can point up or down and can even be split left and right (as in the second plane shown). This picture just shows the common modern shapes; fantasy designs can have multiple wings (vertically stacked or horizontally spaced). Even cylinders can be used as winglets or combined horizontal and vertical stabilizers.

No matter what type of design we choose, there is one key requirement: ***balance of forces and moments***. We covered some general information about balancing and keeping stable; but we did not discuss how to determine CG and ***CL*** (C***enter of Lift***). We need to determine CL for both the wing(s) and horizontal stabilizer(s). Then we need to figure the "best" distance from the CG to the wing(s)' CF and to the horizontal stabilizer(s)' CF. We don't have to worry so much about the wing area (as long as there is plenty for required lift) because planes "self adjust" the angle of attack to make combined lift match total weight. However, we must find the best location and size for the horizontal stabilizer. A big tail closer to the CG and a small tail at a proportionally shorter distance produce the same stabilizing moment; so which is best? So where do we start? Card stock design is similar in some ways to real aircraft design, and totally different in others. So let's go through the process:

Step 1: Choose a wing shape. This is not totally unlike real design, because real design starts with plane function (fighter, small private passenger, large commercial, etc) which can lead to past successful wing shapes. This comes down to choosing either a conventional shape wing or a delta style wing.

If you choose a delta shape or delta combo shape similar to any of the ones shown below, there is less design involved:
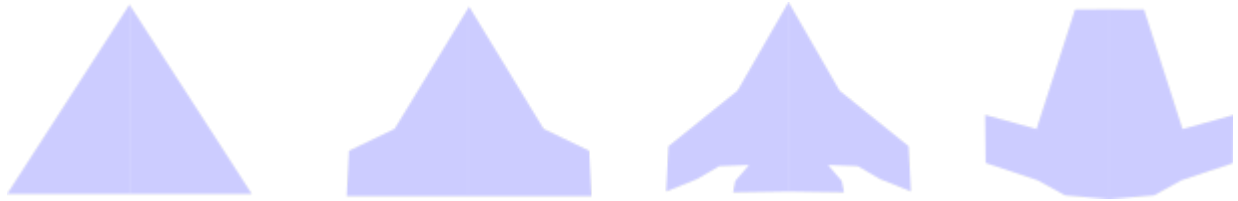
These wing shapes are modeled after high speed aircraft such as fighters. Their flight is more similar to folded paper airplanes than it is to miniaturized real aircraft. Their design is simple: glue wing flat and level on a fuselage. Then, add (or subtract) nose weight until CG is at approximately 30% to 50% of the distance from wing front to wing back as shown:

Add Nose Weight to Get
CG at Near 30% L_W

Wing Flat and Level

C
G

30%
L_W

L_W
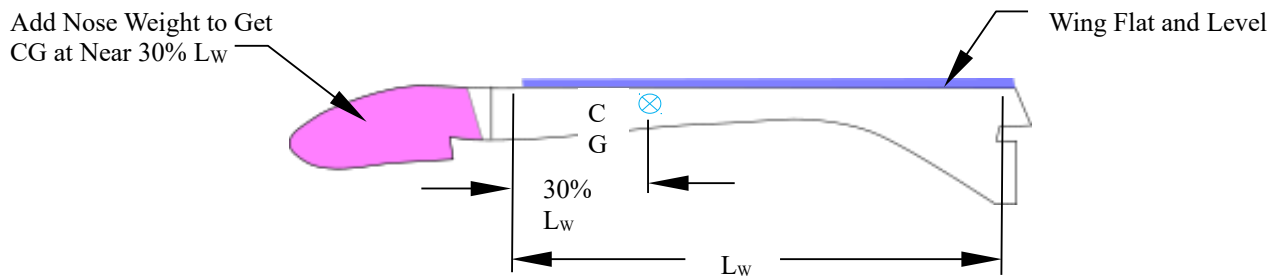
That is pretty much it for delta style wings. Conventional shaped wings are not so simple because we are actually simulating miniaturized versions of real aircraft. So on to Step 2.

Step 2: Choose your wing shape, then determine its **MAC (Mean Aerodynamic Chord)** as follows:

C_F C_T ½ C_T

Equivalent
Wing Tip Edge

MA
C

C_{MAC}

Fuselage
C_L

Fuselage
C_L

½
C_F

C_F C_T
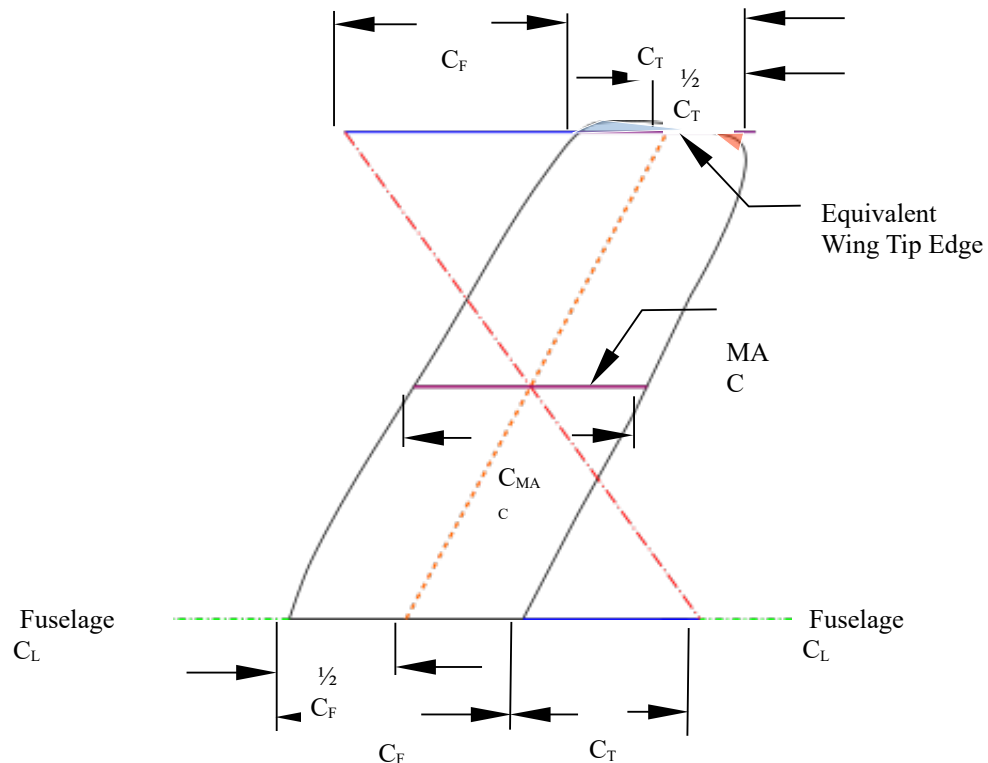
To calculate the MAC location and length, first we must (if wing tip is not straight and parallel to fuselage) construct an equivalent wing tip parallel to fuselage as shown. We do this by trying to construct a line that has as much area above the line as below the line, as shown. Next we construct a line from the midpoint of this line to the midpoint of the cord at the fuselage. Next we construct a line the same length as the cord at the tip just behind fuselage chord and a line the length of the fuselage chord in front of wing tip chord. Then we draw a line from the back of the combined chord at the fuselage to the front of the combined chord at the wing tip. Where this line crosses the line drawn from the midpoint of each chord marks where the MAC is located and all we need to do is draw our MAC by constructing it from the leading edge to the trailing edge through this crossing point. This works for rectangular, elliptical, tapered (single or double) and swept wings. It does not work for combos such as rectangular and tapered or swept back. We will discuss what to do for combos later. Note: swept forward wings are done the same way, only mirrored to swept back wings (most common for canard aircraft).

Step 3: We must now decide what type of airplane we want. As I have mentioned before, Dr. Yasuaki Ninomiya, designer of the famous white wings airplanes, described building two different types of glued card stock airplanes. He described how to design both of these styles in his booklet that came with the set. I bought a set back in the early 1990's, but did not use this information until the last 10 years when I decided to try to design my own glued stock planes. Although my method differs a little, it is highly influenced by his method.

Dr. Ninomiya suggested two styles of design; long distance craft and long duration craft. Long Duration planes use an unconventional design method to gain as much altitude as possible; but this risks instability. They have the tendency to nose up and climb, and are only suitable for outdoor use on a calm day using rubber band launch. The Long Distance uses the design method most real planes use. They tend to fly fast, straight, and level. Long Distance can be used indoors as hand tossed or outdoors with a rubber band launcher. Now that we have decided on which type, let's go back to our MAC diagram:

15% to 25% of C_{MAC}

70% to 80% of C_{MAC}

Fuselage C_L

Fuselage C_L

Long Distance

Long Duration
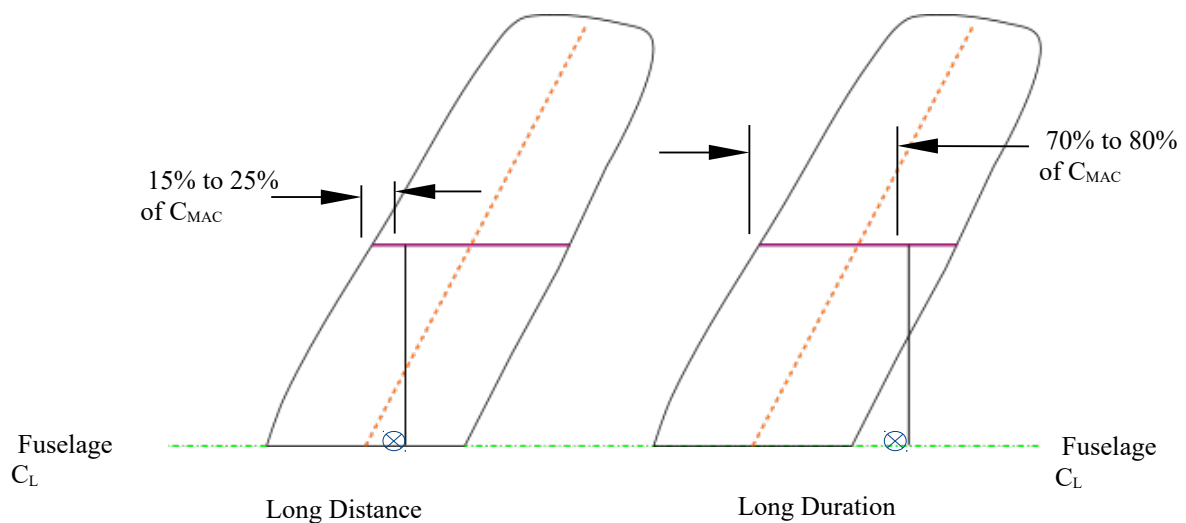
The first difference between the two is the location of the airplane's CG relative to the MAC. For Long Distance type measure approximately 20% of C_{MAC} back from the leading edge of the wing at the MAC. Draw a vertical line down to the fuselage centerline and this locates the intended CG of the airplane. For Long Duration type measure approximately 75% of C_{MAC} back from the leading edge of the wing at the MAC. Draw a vertical line down to the fuselage centerline and this locates the intended CG of the airplane.

Step 4: Next we need to look at the mounting and size.
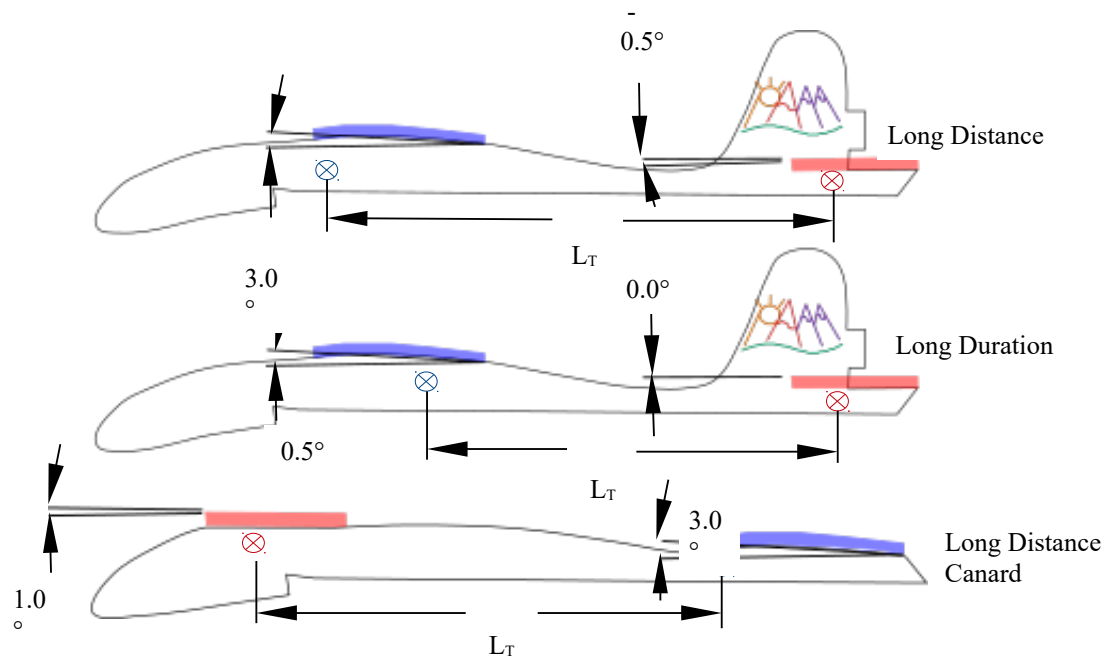
The above illustration shows the angles to mount both wing and horizontal stabilizer for the Long Distance, the Long Duration, and a typical Canard (which is not a good candidate for Long Duration). The value of L_T is set by the designer and we calculate the required area of the horizontal stabilizer as follows:

Long Distance
$$A_{HS} = \frac{0.6 \times A_W \times C_{MAC}}{L_{HS}}$$

Long Duration
$$A_{HS} = \frac{1.2 \times A_W \times C_{MAC}}{L_{HS}}$$

Both
$$A_{VS} = \frac{0.05 \times A_W \times S_W}{L_{VS}}$$

Note: In the figure above, no vertical stabilizer is shown for the canard. But, in reality one is needed. Its area can be split in two and attached as winglets, which were not shown.

Where:

A_{HS} – Area of Horizontal Stabilizer Required

A_W – Calculated Area of Wing

C_{MAC} – Length of MAC

L_{HS} – Distance Between CG of Aircraft and CG of Horizontal Stabilizer

A_{VS} – Area of Vertical Stabilizer Required

L_{VS} – Distance Between CG of Aircraft and CG of Vertical Stabilizer

To avoid some confusion let's summarize the differences in plane styles:

Long Distance:

CG at 15% to 25% of C_{MAC}

Wing at +3.0° Horizontal Stabilizer at -0.5°

$$A_{HS} = \frac{0.6 \times A_W \times C_{MAC}}{L_{HS}}$$

$$A_{VS} = \frac{0.05 \times A_W \times S_W}{L_{VS}}$$

Long Duration:

CG at 70% to 80% of C_{MAC}
Wing at +0.5° Horizontal Stabilizer at 0.0°

$$A_{HS} = \frac{1.2 \times A_W \times C_{MAC}}{L_{HS}}$$

$$A_{VS} = \frac{0.05 \times A_W \times S_W}{L_{VS}}$$

Canard same as Long Distance except:

CG at 15% to 25% of C_{MAC}
Wing at +3.0°
Horizontal Stabilizer at +1.0°

Note: The above mounting angles and tail areas are per Dr. Yasuakı Nınomıya´s suggestıons.

Step 5: Getting the CG at the correct point is the final step. The easiest method is by approximating existing designs and using trial and error. The second method is to use a software like InkScape (it's free) to help design and calculate it. The last is to calculate totally by hand. Since most people would want to take one of the first two options; I will leave the trial and error people to their task and discuss the InkScape option.

InkScape is the software I prefer to design my planes. I even use it to construct my diagrams for folded planes. It is available as either an installed version or a portable program that will run from a memory stick (available from PortableApps.com), allowing you to run even on a library computer. I am not going to write a tutorial on using InkScape here (a later project, maybe) but am just showing how to let it calculate a CG. Note: At the time of this writing there are issues in the InkScape extension where it must be run several times to work.
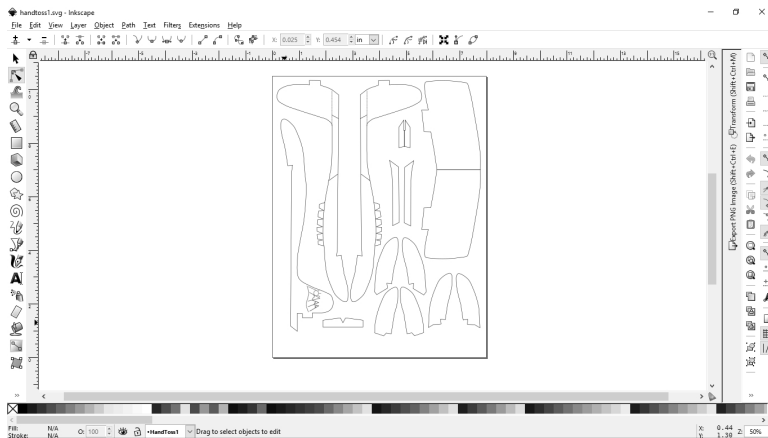

First use InkScape to make an initial design. Although I am not giving a tutorial, I do want to give a couple of tips. Use copy and paste in place then mirror to get left and right pieces. Also, use copy and paste in place to make a copy of fuselage and then delete points to get nose weights, and other body parts that need to match the center fuselage profile. Experiment with group and ungroup along with rotating objects to move parts around to fit on page. Learn to use the snap to commands (like midpoint, cusp point, and intersection). Last, use transform object and the X & Y coordinate indicators to precisely move pieces in place (such as two wing halves).
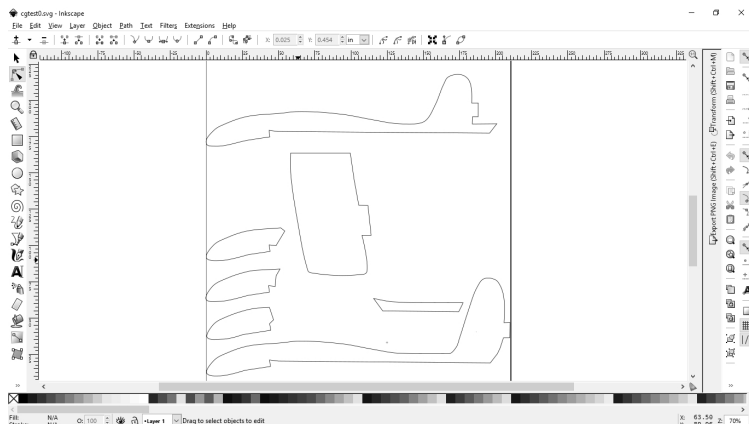
Now copy and paste one of each type of aircraft parts into a new drawing. Rotate so every part is located and rotated to match how it is to be glued to form the aircraft. Each part needs to be correctly located from the left edge fairly closely, but perfection is not required.
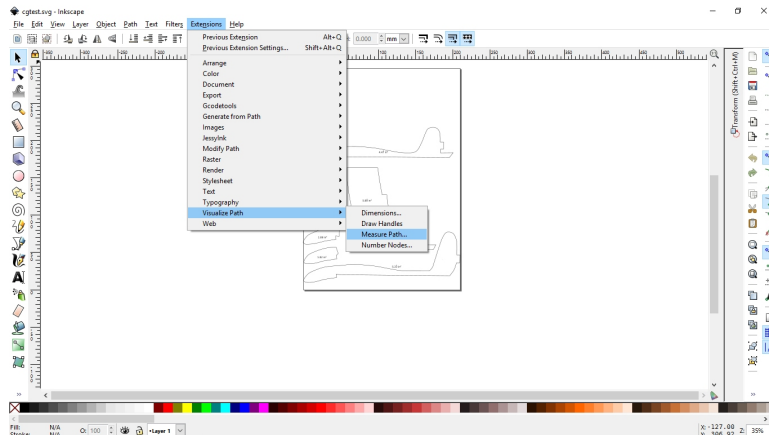
Now under extensions, select Visualize Path and then select Measure Path.
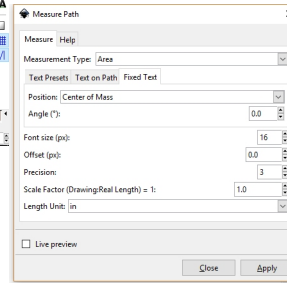
When the pop up comes up, select Measurement Type as Area and Position as Center of Mass. Highlight an object and hit apply. Continue doing this for each item. Then select Measurement Type as Center of Mass and select an object and hit apply. This extension does not always work at the time of this writing and each item may require multiple tries. If you do not see text appear near the center of the object, then select another item and re select the first item. If the selection window (boundary line surrounding object) extends past border, the program extension put text in wrong location and it needs to be deleted and tried again. When all the areas appear to have an area value near the center, then try to get at least one point by selecting Measurement Type Center of Mass. This point can be copied and pasted just above and to the right of the number portion of area text. Use the one point as a guide on approximately where to copy to each area text. Hopefully, this issue will be resolved soon.
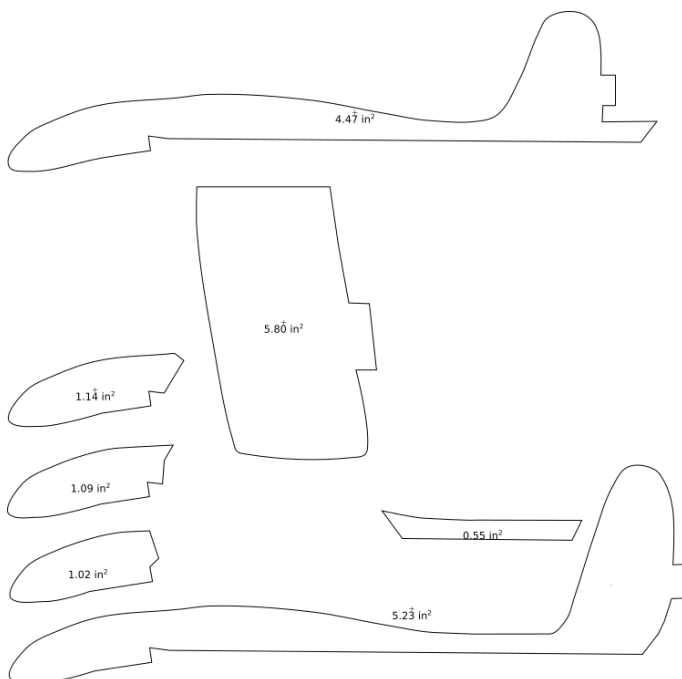
You should end up with something like this diagram.

Now we will discuss a technique where we take an average; not just a standard average, but an average that is weighted to a particular variable. For calculating a composite CG for example, it is weighted to the area of each piece as shown by the following equation:

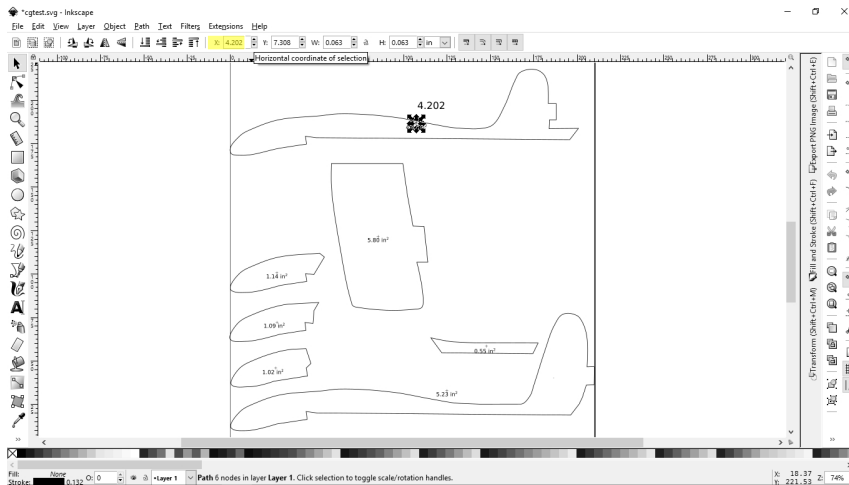$$\overline{X}_{TOTAL} = \frac{\sum \overline{x}\, A_n \times q}{\sum A_n \times q}$$

Where:

$\overline{X}_{TOTAL} = CG_{X\,Direction}$

$\sum \overline{x}\, A_n = The\ sum\ of\ each\ \ CG_{X\,Direction}$
times $it's\ area$

$\sum A_n = The \sum of\ all\ the\ area's$

q = quantity of that area (ex.: 2 if a left and right hand piece)

To get the CG$_{X\,Direction}$ for each piece:

Select the point (cross hair) just above the area for each piece and read the x position (highlighted in yellow). Select the insert text and type this value near the area text for each piece. You will end up with a diagram as shown below:
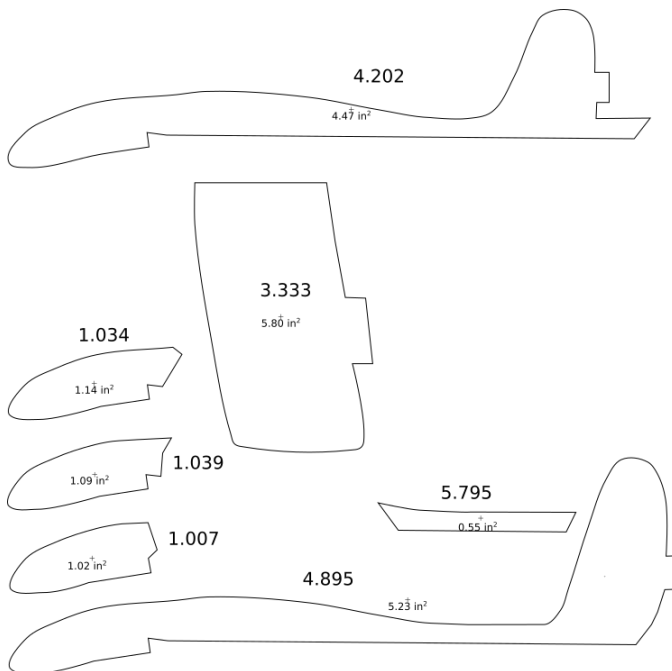

Now we just calculate from the above equation for $\overline{X}_{TOTAL}$

I prefer to make this easy by using a spread sheet like LibreOffice Calc.

Part #	Quantity	X Bar	Area	A * Q	X Bar * A * Q
1	1	4.202	4.47	4.47	18.78294
2	2	3.333	5.8	11.6	38.6628
3	2	4.895	5.23	10.46	51.2017
4	2	1.034	1.14	2.28	2.35752
5	2	1.039	1.09	2.18	2.26502
6	2	1.007	1.02	2.04	2.05428
7	2	5.795	0.55	1.1	6.3745
				34.13	121.69876

X Bar Total =   3.5657415763

Since I designed this plane trial and error let's look at how close I got the CG. It looks like I made it more like a Long Duration aircraft. I might consider moving the CG forward by adding more nose weight to gain performance as a hand toss design.
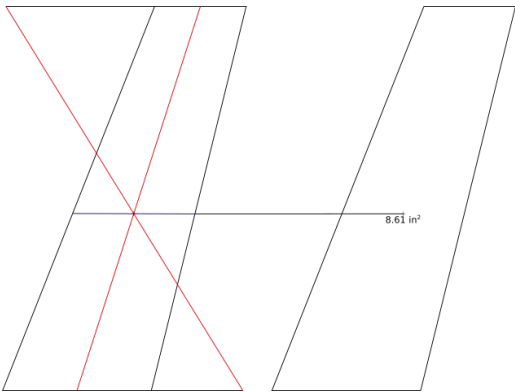
Example of how to find C$_{MAC}$ for combo wing

1.5

3.594
1.87 in^2

C$_{MA}$

1
C
4.50 ...

2.0
9
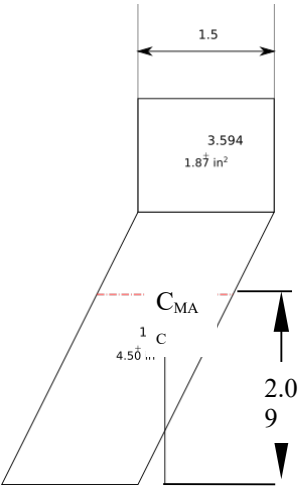
The C$_{MAC}$ of a combo wing can be found by the same weighted average method as CG. The CG can be found by measuring 15% to 25% of C$_{MAC}$ (Long Distance) or 70% to 80% (Long Duration).

Part #	Y Bar	Area	YBar * A
1	3.594	1.87	6.72078
2	1.469	4.5	6.6105
		6.37	13.33128

Y Bar Total =   2.092822606

8.61 in^2

Remember the taper wing diagram on the far right? The one where we kept constructing lines? When we compare to the diagram on the left, we find that in reality what we had done with construction lines is find the CG of a tapered body. From this we learn three things. First, we learn that using InkScape for a combo wing can include tapered sections. Second, this is because the C$_{MAC}$ passes through the Y bar portion of the half wing centroid. Last, we can use this to show we should get a very close location of the C$_{MAC}$ if we use InkScape to find the CG of a wing with curved edges. Note: If you have a taper as part of combo wing, you must use a weighted average to find the length C$_{MAC}$, not just the location as shown below:

1.25 in
1.25 in^2

2.25 in^2
1.51 in

1.75 in   4.37 in^2
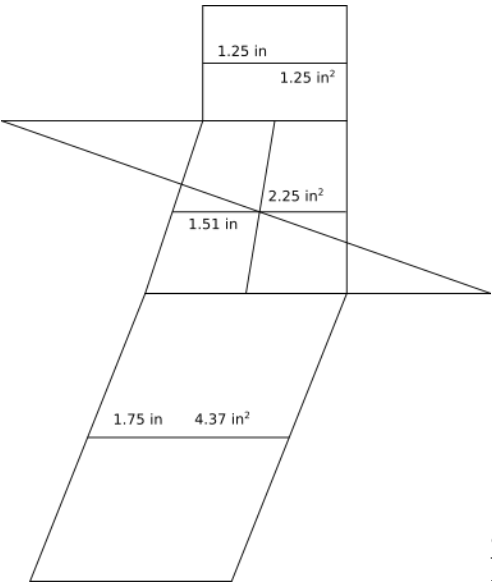
| Part #	L MAC	Area	L MAC * A
1	1.25	1.25	1.5625
2	1.51	2.25	3.3975
3	1.75	4.37	7.6475
		7.87	12.6075

L MAC AVG = 1.6019695044

I have found that often InkScape will calculate an accurate area and line length, but not get an accurate CG. When this happens I use the construction method to find the CG of the tapered section. The CG of the non tapered sections is located at the center of the middle line.
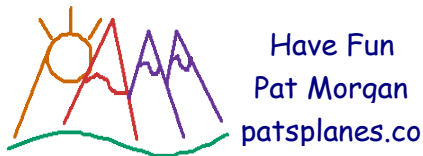
Just a few more tips:

If you are looking for slower gliding flight like Long Duration, use a long narrow wing (high aspect ratio) even if it requires a second layer of the wing for strength.

Set dihedral angle for high wing at about 10°, mid wing at about 15°, and low wing at about 20°.

Wing camber (amount of curve height) is 4% to 5% of C_{MAC}:

4% to 5%
C_{MAC}
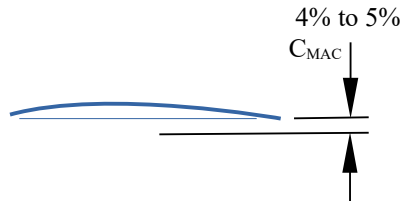
This concludes the design of glued card stock airplanes. I hope it was understandable and useful to the serious science student. It might seem complex, but is actually just a long process. However, the sense of accomplishment you get when you get a nice flight the first time by running the calculations can be well worth it. Especially as a science project that is fun!

Have Fun
Pat Morgan
patsplanes.co

The cool paper airplane site!